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Periodic solutions for a delayed predator-prey model of prey dispersal in two-patch environments

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Abstract

A delayed periodic Lotka–Volterra type predator-prey model with prey dispersal in two-patch environments is investigated. By using Gaines and Mawhin's continuation theorem of coincidence degree theory and by means of a suitable Lyapunov functional, a set of easily verifiable sufficient conditions are obtained to guarantee the existence, uniqueness and global stability of positive periodic solutions of the system. Numerical simulations are given to illustrate the feasibility of our main results.

Keywords: Dispersion; Time delay; Periodic solution; Persistence; Global stability

1. Introduction

The effect of environment change in the growth and diffusion of a species in a heterogeneous habitat is a subject of considerable interest in the ecological literature. Since the pioneering theoretical work by Skellam [28], many works have focused on the effect of spatial factors which play a crucial rule in the persistence and stability of a population ([1–5,8–12,19,21–24,28–34,36–39], and references cited therein). Many of the existing models deal with a single population dispersing among patches. Some of them deal with competition and predator-prey interactions in patchy environments. The analyses of these models have been centered around the coexistence of populations and

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the stability of equilibria. These works indicate that a diffusion process in an ecological system is often considered to have a stabilizing influence on the system [19,31], but is also probably destabilizing the system [22,23]. Kuang and Takeuchi [21] proposed and discussed the following predator-prey system in a two-patch environment:

$$\begin{aligned}
\dot{x}_1 &= x_1 g_1(x_1) - y p_1(x_1) + \varepsilon(x_2 - x_1),\\
\dot{x}_2 &= x_2 g_2(x_2) - y p_2(x_2) + \varepsilon(x_1 - x_2),\\
\dot{y} &= y[-s(y) + c_1 p_1(x_1) + c_2 p_2(x_2)],\\
x_i(0) &\geqslant 0, y(0) \geqslant 0, i = 1,2,
\end{aligned} \tag{1.1}$$

where $x_i(t)$ represents the prey population in the ith patch, $i = 1,2$, at time $t \geqslant 0$. $y(t)$ stands for the total predator population for both patches. The predator population is assumed to have no barriers between patches. $g_i(x_i)$ is the specific growth rate for the prey population in the absence of predation when it is restricted to the i-th patch. $p_i(x_i)$ is the predator functional response of the predator population on the prey in the i-th patch. ε is a positive constant that can be viewed as the dispersal rate or inverse barrier strength. $s(y)$ is the density-dependent death rate of the predator in the absence of prey. $c_i > 0$ is the conversion ratio of prey into predator. Conditions have been established in [21] for the existence, uniform persistence, and local and global stability of positive steady states of system (1.1).

In most of the models of populations dispersing among patches in a heterogeneous environment considered so far, it has been assumed that all biological and environmental parameters are constants in time. However, any biological or environmental parameters are naturally subject to fluctuation in time. The effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Thus, the assumptions of periodicity of the parameters are a way of incorporating the periodicity of the environment (such as seasonal effects of weather, food supplies, mating habits and so forth).

On the other hand, time delays of one type or another have been incorporated into biological models by many researchers, we refer to the monographs of Cushing [7], Gopalsamy [16], Kuang [20], and MacDonald [25] for general delayed biological systems and to Beretta and Kuang [6], Gopalsamy [14,15], Hastings [18], May [26], Ruan [27], Wang [35] and the references cited therein for studies on delayed biological systems. In general, delay differential equations exhibit much more complicated dynamics than ordinary differential equations since a time delay could cause a stable equilibrium to become unstable and cause the population to fluctuate. Time delay due to gestation is a common example, because generally the consumption of prey by the predator throughout its past history governs the present birth rate of the predator. Therefore, more realistic models of population interactions should take into account the seasonality of the changing environment, dispersal and the effect of time delays.

Motivated by the work of Kuang and Takeuchi in [21], in the present paper we are concerned with the study on the combined effects of dispersion, periodicity of the environment and time delays on the dynamics of predator-prey systems. To do so we discuss the following delayed periodic Lotka–Volterra type predator-prey system with prey dispersal in two-patch environments

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{13}(t)y(t)] + D_1(t)(x_2(t) - x_1(t)),\\ \dot{x}_2(t) &= x_2(t)[r_2(t) - a_{22}(t)x_2(t) - a_{23}(t)y(t)] + D_2(t)(x_1(t) - x_2(t)),\\ \dot{y}(t) &= y(t)[-r_3(t) + a_{31}(t)x_1(t-\tau_1) + a_{32}(t)x_2(t-\tau_1) - a_{33}(t)y(t-\tau_2)],\end{aligned} \tag{1.2}$$

with initial conditions

$$\begin{aligned}&x_i(\theta) = \phi_i(\theta), y(\theta) = \psi(\theta), \theta \in [-\tau, 0],\\ &\phi_i(0) > 0, \psi(0) > 0, \phi_i, \psi \in C([-\tau, 0), R_+), i = 1, 2,\end{aligned} \tag{1.3}$$

where $x_i(t)$ denotes the density of species x in patch $i, i = 1, 2$, at time t, and $y(t)$ denotes the total predator population for both patches. $r_i(t)$ is the intrinsic growth rate of the prey in patch $i, i = 1, 2$; $a_{ii}(t)(i = 1, 2)$ are the density-dependent coefficients of the prey at patch i; $a_{13}(t)$ and $a_{23}(t)$ are the capturing rates of the predator in patch 1 and 2, respectively, $a_{31}(t)/a_{13}(t)$ and $a_{32}(t)/a_{23}(t)$ are the conversion rates of nutrients into the reproduction of the predator, $r_3(t)$ is the death rate of the predator; $D_i(t)$ is the dispersion rate of prey species $x, i = 1, 2$. $\tau = \max\{\tau_1, \tau_2\}$. τ_1 is the delay due to gestation, that is, mature adult predators can only contribute to the production of predator biomass. In addition, we have included the term $-a_{33}y(t-\tau_2)$ in the dynamics of the predator to incorporate the negative feedback of predator crowding.

In this paper, for system (1.2) we always assume that:

(H1) $r_i(t), a_{ij}(t)(i,j=1,2,3), D_1(t)$ and $D_2(t)$ are continuously positive periodic functions with period ω; τ_1 and τ_2 are nonnegative constants.

It is well known by the fundamental theory of functional differential equations [17] that system (1.2) has a unique solution $z(t) = (x_1(t), x_2(t), y(t))$ satisfying initial conditions (1.3). It is easy to verify that solutions of system (1.2) corresponding to initial conditions (1.3) are defined on $[0, +\infty)$ and remain positive for all $t \geqslant 0$. In this paper, the solution of system (1.2) satisfying initial conditions (1.3) will be called to be positive.

The organization of this paper is as follows. In the next section, by using Gaines and Mawhin's continuation theorem of coincidence degree theory, we discuss the existence of positive ω-periodic solutions of system (1.2). By means of a suitable Lyapunov functional, a set of easily verifiable sufficient conditions are derived for the uniqueness and global stability of the positive periodic solutions of system (1.2) in Section 3. Finally, numerical simulations are presented to illustrate the feasibility of our main results.

2. Existence of periodic solutions

In this section, by using Gaines and Mawhin's continuation theorem, we establish sufficient conditions for the existence of positive periodic solutions of (1.2)–(1.3). To this end, we first summarize in the following a few concepts and results from [13] that will be basic for this section.

Let X, Y be real Banach spaces, L: $\mathrm{Dom}L \subset X \to Y$ a linear mapping, and $N : X \to Y$ a continuous mapping. The mapping L is called a Fredholm mapping of index zero if $\dim \mathrm{Ker}\,L = \mathrm{codim}\ \mathrm{Im}\,L < +\infty$ and $\mathrm{Im}\,L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$, and $Q : Y \to Y$ such that $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Ker}\,Q = \mathrm{Im}\,L = \mathrm{Im}(I - Q)$, then the restriction L_P of L to $\mathrm{Dom}L \cap \mathrm{Ker}P : (I - P)X \to \mathrm{Im}\,L$ is invertible. Denote the inverse of L_P by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Lemma 2.1. *Let $\Omega \subset X$ be an open bounded set. Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Assume*

(a) *for each* $\lambda \in (0, 1), x \in \partial\Omega \cap \mathrm{Dom}L$, $Lx \neq \lambda Nx$;
(b) *for each* $x \in \partial\Omega \cap \mathrm{Ker}L$, $QNx \neq 0$;
(c) $\deg\{JQN, \Omega \cap \mathrm{Ker}L, 0\} \neq 0$.

Then $Lx = Nx$ has at least one solution in $\bar{\Omega} \cap \mathrm{Dom}\,L$.

In what follows we shall use the notations:

$$\bar{f} = \frac{1}{\omega}\int_0^{\omega} f(t)\,\mathrm{d}t, \qquad f^L = \min_{t\in[0,\omega]} f(t), \qquad f^M = \max_{[0,\omega]} f(t),$$

where f is a continuous ω-periodic function.

We are now in a position to formulate our result on the existence of periodic solutions of system (1.2).

Theorem 2.1. *In addition to* (H1), *assume further that*

(H2) $a_{33}^L\overline{(r_1 - D_1)} - a_{13}^M A(a_{31}^M + a_{32}^M) > 0$,
(H3) $a_{33}^L\overline{(r_2 - D_2)} - a_{23}^M A(a_{31}^M + a_{32}^M) > 0$,
(H4) $a_{31}^L a_{22}^M\overline{(r_1 - D_1)} + a_{32}^L a_{11}^M\overline{(r_2 - D_2)} - \overline{r_3} a_{11}^M a_{22}^M > 0$,

where

$$A = \max\left\{\frac{(r_1 - D_1)^M + D_1^M}{a_{11}^L}, \quad \frac{(r_2 - D_2)^M + D_2^M}{a_{22}^L}\right\}.$$

Then system (1.2) *has at least one positive ω-periodic solution.*

Proof. Making the change of variables

$$u_1(t) = \ln\{x_1(t)\}, \quad u_2(t) = \ln\{x_2(t)\}, \quad u_3(t) = \ln\{y(t)\}, \tag{2.1}$$

then (1.2) becomes

$$\begin{aligned}
\frac{\mathrm{d}u_1(t)}{\mathrm{d}t} &= r_1(t) - D_1(t) - a_{11}(t)\mathrm{e}^{u_1(t)} - a_{13}(t)\mathrm{e}^{u_3(t)} + D_1(t)\mathrm{e}^{u_2(t)-u_1(t)},\\
\frac{\mathrm{d}u_2(t)}{\mathrm{d}t} &= r_2(t) - D_2(t) - a_{22}(t)\mathrm{e}^{u_2(t)} - a_{23}(t)\mathrm{e}^{u_3(t)} + D_2(t)\mathrm{e}^{u_1(t)-u_2(t)},\\
\frac{\mathrm{d}u_3(t)}{\mathrm{d}t} &= -r_3(t) + a_{31}(t)\mathrm{e}^{u_1(t-\tau_1)} + a_{32}(t)\mathrm{e}^{u_2(t-\tau_1)} - a_{33}(t)\mathrm{e}^{u_3(t-\tau_2)}.
\end{aligned} \tag{2.2}$$

Obviously, if system (2.2) has one ω-periodic solution $(u_1^*(t), u_2^*(t), u_3^*(t))^{\mathrm{T}}$, then $z^*(t) = (x_1^*(t), x_2^*(t), y^*(t))^{\mathrm{T}} = (\exp\{u_1^*(t)\}, \exp\{u_2^*(t)\}, \exp\{u_3^*(t)\})^{\mathrm{T}}$ is a positive ω-periodic solution of system (1.2). Hence, to complete the proof, it suffices to verify that system (2.2) has at least one ω-periodic solution.

Set

$$X = Y = \left\{(u_1(t), u_2(t), u_3(t))^{\mathrm{T}} \in C(R, R^3) : u_i(t+\omega) = u_i(t), i = 1, 2, 3\right\}$$

and

$$\|(u_1(t), u_2(t), u_3(t))^{\mathrm{T}}\| = \sum_{i=1}^{3} \max_{t\in[0,\omega]} |u_i(t)|,$$

here $|\cdot|$ denotes the Euclidean norm. Then X and Y are both Banach spaces when they are endowed with the norm $\|\cdot\|$.

Let

$$L : \mathrm{Dom}\, L \cap X, \quad L(u_1(t), u_2(t), u_3(t))^{\mathrm{T}} = \left(\frac{\mathrm{d}u_1(t)}{\mathrm{d}t}, \frac{\mathrm{d}u_2(t)}{\mathrm{d}t}, \frac{\mathrm{d}u_3(t)}{\mathrm{d}t}\right)^{\mathrm{T}},$$

where $\mathrm{Dom}\, L = \{(u_1(t), u_2(t), u_3(t))^{\mathrm{T}} \in C^1(R, R^3)\}$ and $N : X \to X$,

$$N \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} r_1(t) - D_1(t) - a_{11}(t)\mathrm{e}^{u_1(t)} - a_{13}(t)\mathrm{e}^{u_3(t)} + D_1(t)\mathrm{e}^{u_2(t)-u_1(t)} \\ r_2(t) - D_2(t) - a_{22}(t)\mathrm{e}^{u_2(t)} - a_{23}(t)\mathrm{e}^{u_3(t)} + D_2(t)\mathrm{e}^{u_1(t)-u_2(t)} \\ -r_3(t) + a_{31}(t)\mathrm{e}^{u_1(t-\tau_1)} + a_{32}(t)\mathrm{e}^{u_2(t-\tau_1)} - a_{33}(t)\mathrm{e}^{u_3(t-\tau_2)} \end{bmatrix}.$$

Define

$$P \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = Q \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_0^{\omega} u_1(t)\,\mathrm{d}t \\ \frac{1}{\omega}\int_0^{\omega} u_2(t)\,\mathrm{d}t \\ \frac{1}{\omega}\int_0^{\omega} u_3(t)\,\mathrm{d}t \end{bmatrix}, \qquad \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \in X = Y.$$

Obviously, we have

$$\mathrm{Ker}\, L = \{x| \quad x \in X, \quad x = h, \quad h \in R^3\},$$

$$\mathrm{Im}\, L = \left\{y| \quad y \in Y, \quad \int_0^{\omega} y(t)\,\mathrm{d}t = 0\right\},$$

and

$$\dim\ \mathrm{Ker}\, L = \mathrm{codim}\ \mathrm{Im}\, L = 3.$$

Since $\mathrm{Im}\, L$ is closed in Y, L is a Fredholm mapping of index zero. It is easy to show that P and Q are continuous projectors such that

$$\mathrm{Im}\, P = \mathrm{Ker}\, L, \quad \mathrm{Ker}\, Q = \mathrm{Im}\, L = \mathrm{Im}\,(I - Q).$$

Furthermore, through an easy computation, we can verify that the inverse K_P of L_P has the form

$$K_P : \mathrm{Im}\, L \to \mathrm{Dom}\, L \cap \mathrm{Ker}\, P,$$

$$K_P(y) = \int_0^t y(s)\,\mathrm{d}s - \frac{1}{\omega}\int_0^{\omega}\int_0^t y(s)\,\mathrm{d}s\,\mathrm{d}t.$$

Accordingly, $QN : X \to Y$ and $K_P(I-Q)N : X \to X$ read

$$QNx = \begin{bmatrix} \dfrac{1}{\omega}\displaystyle\int_0^{\omega} [r_1(t) - D_1(t) - a_{11}(t)\mathrm{e}^{u_1(t)} - a_{13}(t)\mathrm{e}^{u_3(t)} + D_1(t)\mathrm{e}^{u_2(t)-u_1(t)}]\,\mathrm{d}t \\ \dfrac{1}{\omega}\displaystyle\int_0^{\omega} [r_2(t) - D_2(t) - a_{22}(t)\mathrm{e}^{u_2(t)} - a_{23}(t)\mathrm{e}^{u_3(t)} + D_2(t)\mathrm{e}^{u_1(t)-u_2(t)}]\,\mathrm{d}t \\ \dfrac{1}{\omega}\displaystyle\int_0^{\omega} [-r_3(t) + a_{31}(t)\mathrm{e}^{u_1(t-\tau_1)} + a_{32}(t)\mathrm{e}^{u_2(t-\tau_1)} - a_{33}(t)\mathrm{e}^{u_3(t-\tau_2)}]\,\mathrm{d}t \end{bmatrix},$$

$$K_P(I-Q)Nx = \int_0^t Nx(s)\,\mathrm{d}s - \frac{1}{\omega}\int_0^{\omega}\int_0^t Nx(s)\,\mathrm{d}s\,\mathrm{d}t - \left(\frac{t}{\omega} - \frac{1}{2}\right)\int_0^{\omega} Nx(s)\,\mathrm{d}s.$$

Obviously, QN and $K_P(I-Q)N$ are continuous by the Lebesgue theorem. Moreover, $\overline{QN(\bar{\Omega})}$ and $\overline{K_P(I-Q)N(\bar{\Omega})}$ are relatively compact for any open bounded set $\Omega \subset X$. Therefore, N is L-compact on $\bar{\Omega}$ for any open bounded set $\Omega \subset X$.

In order to apply Lemma 2.1, we need to search for a suitable open, bounded subset Ω.

Corresponding to the operator equation $Lx = \lambda Nx, \lambda \in (0,1)$, one has

$$\begin{aligned} \frac{\mathrm{d}u_1(t)}{\mathrm{d}t} &= \lambda\left[r_1(t) - D_1(t) - a_{11}(t)\mathrm{e}^{u_1(t)} - a_{13}(t)\mathrm{e}^{u_3(t)} + D_1(t)\mathrm{e}^{u_2(t)-u_1(t)}\right], \\ \frac{\mathrm{d}u_2(t)}{\mathrm{d}t} &= \lambda\left[r_2(t) - D_2(t) - a_{22}(t)\mathrm{e}^{u_2(t)} - a_{23}(t)\mathrm{e}^{u_3(t)} + D_2(t)\mathrm{e}^{u_1(t)-u_2(t)}\right], \\ \frac{\mathrm{d}u_3(t)}{\mathrm{d}t} &= \lambda\left[-r_3(t) + a_{31}(t)\mathrm{e}^{u_1(t-\tau_1)} + a_{32}(t)\mathrm{e}^{u_2(t-\tau_1)} - a_{33}(t)\mathrm{e}^{u_3(t-\tau_2)}\right]. \end{aligned} \tag{2.3}$$

Suppose that $(u_1(t), u_2(t), u_3(t))^{\mathrm{T}} \in X$ is a solution of (2.3) for a certain $\lambda \in (0,1)$. Integrating (2.3) over the interval $[0,\omega]$ leads to

$$\int_0^\omega a_{11}(t)\mathrm{e}^{u_1(t)}\,\mathrm{d}t + \int_0^\omega a_{13}(t)\mathrm{e}^{u_3(t)}\,\mathrm{d}t = \int_0^\omega (r_1(t) - D_1(t))\,\mathrm{d}t + \int_0^\omega D_1(t)\mathrm{e}^{u_2(t)-u_1(t)}\,\mathrm{d}t, \tag{2.4}$$

$$\int_0^\omega a_{22}(t)\mathrm{e}^{u_2(t)}\,\mathrm{d}t + \int_0^\omega a_{23}(t)\mathrm{e}^{u_3(t)}\,\mathrm{d}t = \int_0^\omega (r_2(t) - D_2(t))\,\mathrm{d}t + \int_0^\omega D_2(t)\mathrm{e}^{u_1(t)-u_2(t)}\,\mathrm{d}t, \tag{2.5}$$

$$\int_0^\omega a_{31}(t)\mathrm{e}^{u_1(t-\tau_1)}\,\mathrm{d}t + \int_0^\omega a_{32}(t)\mathrm{e}^{u_2(t-\tau_1)}\,\mathrm{d}t = \int_0^\omega r_3(t)\mathrm{d}t + \int_0^\omega a_{33}(t)\mathrm{e}^{u_3(t-\tau_2)}\,\mathrm{d}t. \tag{2.6}$$

Multiplying the first equation of (2.3) by $\mathrm{e}^{u_1(t)}$ and integrating over $[0,\omega]$ gives

$$\int_0^\omega a_{11}(t)\mathrm{e}^{2u_1(t)}dt < \int_0^\omega (r_1(t) - D_1(t))\mathrm{e}^{u_1(t)}\,\mathrm{d}t + \int_0^\omega D_1(t)\mathrm{e}^{u_2(t)}\,\mathrm{d}t,$$

which yields

$$a_{11}^L \int_0^\omega \mathrm{e}^{2u_1(t)}\,\mathrm{d}t < (r_1 - D_1)^M \int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t + D_1^M \int_0^\omega \mathrm{e}^{u_2(t)}\,\mathrm{d}t. \tag{2.7}$$

Similarly, multiplying the second equation in (2.3) by $\mathrm{e}^{u_2(t)}$ and integrating over $[0,\omega]$ gives

$$\int_0^\omega a_{22}(t)\mathrm{e}^{2u_2(t)}\,\mathrm{d}t < \int_0^\omega (r_2(t) - D_2(t))\mathrm{e}^{u_2(t)}\mathrm{d}t + \int_0^\omega D_2(t)\mathrm{e}^{u_1(t)}\,\mathrm{d}t,$$

which implies

$$a_{22}^L \int_0^\omega \mathrm{e}^{2u_2(t)}\,\mathrm{d}t < (r_2 - D_2)^M \int_0^\omega \mathrm{e}^{u_2(t)}\,\mathrm{d}t + D_2^M \int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t. \tag{2.8}$$

By using the inequalities

$$\left(\int_0^\omega \mathrm{e}^{u_i(t)}\,\mathrm{d}t\right)^2 \leqslant \omega \int_0^\omega \mathrm{e}^{2u_i(t)}\,\mathrm{d}t, \qquad i = 1, 2,$$

it follows from (2.7) and (2.8) that

$$a_{11}^L \left(\int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t\right)^2 < \omega(r_1 - D_1)^M \int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t + D_1^M \omega \int_0^\omega \mathrm{e}^{u_2(t)}\,\mathrm{d}t, \tag{2.9}$$

$$a_{22}^L \left(\int_0^\omega \mathrm{e}^{u_2(t)}\,\mathrm{d}t\right)^2 < \omega(r_2 - D_2)^M \int_0^\omega \mathrm{e}^{u_2(t)}\,\mathrm{d}t + D_2^M \omega \int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t. \tag{2.10}$$

If $\int_0^\omega \mathrm{e}^{u_2(t)}\,\mathrm{d}t \leqslant \int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t$, then it follows from (2.9) that

$$a_{11}^L \left(\int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t\right)^2 < \omega(r_1 - D_1)^M \int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t + D_1^M \omega \int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t,$$

which leads to

$$\int_0^{\omega} e^{u_2(t)}\,dt \leqslant \int_0^{\omega} e^{u_1(t)}\,dt < \frac{\omega(r_1-D_1)^M+\omega D_1^M}{a_{11}^L}. \tag{2.11}$$

If $\int_0^{\omega} e^{u_1(t)}\,dt \leqslant \int_0^{\omega} e^{u_2(t)}dt$, then it follows from (2.10) that

$$a_{22}^L\left(\int_0^{\omega} e^{u_2(t)}\,dt\right)^2 < \omega(r_2-D_2)^M\int_0^{\omega} e^{u_2(t)}\,dt + D_2^M\omega\int_0^{\omega} e^{u_2(t)}\,dt,$$

which yields

$$\int_0^{\omega} e^{u_1(t)}\,dt \leqslant \int_0^{\omega} e^{u_2(t)}\,dt < \frac{\omega(r_2-D_2)^M+\omega D_2^M}{a_{22}^L}, \tag{2.12}$$

Set

$$A=\max\left\{\frac{(r_1-D_1)^M+D_1^M}{a_{11}^L},\quad \frac{(r_2-D_2)^M+D_2^M}{a_{22}^L}\right\}. \tag{2.13}$$

Then it follows from (2.11)–(2.13) that

$$\int_0^{\omega} e^{u_i(t)}\,dt < \omega A,\quad i=1,2. \tag{2.14}$$

Since $(u_1(t),u_2(t),u_3(t))^{\mathrm{T}}\in X$, there exist $\xi_i,\eta_i\in[0,\omega]$ such that

$$u_i(\xi_i)=\min_{t\in[0,\omega]}u_i(t),\quad u_i(\eta_i)=\max_{t\in[0,\omega]}u_i(t),\quad i=1,2,3. \tag{2.15}$$

It follows from (2.14) and (2.15) that

$$u_i(\xi_i)<\ln A,\quad i=1,2. \tag{2.16}$$

Noting that

$$\begin{aligned}&\int_0^{\omega} e^{u_i(t-\tau_1)}\,dt=\int_0^{\omega} e^{u_i(t)}\,dt,\quad i=1,2,\\ &\int_0^{\omega} e^{u_3(t-\tau_2)}dt=\int_0^{\omega} e^{u_3(t)}\,dt,\end{aligned} \tag{2.17}$$

we derive from (2.6) and (2.14) that

$$\begin{aligned}\int_0^{\omega} a_{33}(t)e^{u_3(t-\tau_2)}\,dt &\leqslant \int_0^{\omega} a_{31}(t)e^{u_1(t-\tau_1)}\,dt+\int_0^{\omega} a_{32}(t)e^{u_2(t-\tau_1)}\,dt\\ &\leqslant a_{31}^M\int_0^{\omega} e^{u_1(t-\tau_1)}\,dt+a_{32}^M\int_0^{\omega} e^{u_2(t-\tau_1)}\,dt\\ &= a_{31}^M\int_0^{\omega} e^{u_1(t)}\,dt+a_{32}^M\int_0^{\omega} e^{u_2(t)}\,dt\\ &< \omega A\left(a_{31}^M+a_{32}^M\right),\end{aligned}$$

which implies

$$\int_0^{\omega} e^{u_3(t)}\,dt \leqslant \frac{\omega A(a_{31}^M+a_{32}^M)}{a_{33}^L}, \tag{2.18}$$

and

$$u_3(\xi_3) \leqslant \ln\frac{A(a_{31}^M + a_{32}^M)}{\overline{a_{33}}}. \tag{2.19}$$

It follows from (2.3), (2.4), (2.14), (2.17) and (2.18) that

$$\begin{aligned}\int_0^\omega |u_1'(t)|\mathrm{d}t &< \int_0^\omega [r_1(t) - D_1(t) + a_{11}(t)\mathrm{e}^{u_1(t)} \\ &\quad + a_{13}(t)\mathrm{e}^{u_3(t)} + D_1(t)\mathrm{e}^{u_2(t)-u_1(t)}]\,\mathrm{d}t \\ &= 2\int_0^\omega a_{11}(t)\mathrm{e}^{u_1(t)}\,\mathrm{d}t + 2\int_0^\omega a_{13}(t)\mathrm{e}^{u_3(t)}\,\mathrm{d}t \\ &< 2\omega A\left(a_{11}^M + \frac{a_{13}^M(a_{31}^M + a_{32}^M)}{a_{33}^L}\right) \\ &\stackrel{\text{def}}{=} c_1.\end{aligned} \tag{2.20}$$

Similarly, from (2.5), (2.6), (2.14), (2.17) and(2.18), we can derive

$$\begin{aligned}&\int_0^\omega |u_2'(t)|\mathrm{d}t < 2\omega A\left(a_{22}^M + \frac{a_{23}^M(a_{31}^M + a_{32}^M)}{a_{33}^L}\right) \stackrel{\text{def}}{=} c_2, \\ &\int_0^\omega |u_3'(t)|\mathrm{d}t < 2\omega A\left(a_{31}^M + a_{32}^M\right) \stackrel{\text{def}}{=} c_3.\end{aligned} \tag{2.21}$$

Then from (2.16), (2.19), (2.20) and (2.21), we obtain

$$u_i(t) \leqslant u_i(\xi_i) + \int_0^\omega |u_i'(t)|\,\mathrm{d}t < \ln A + c_i, \quad i = 1, 2, \tag{2.22}$$

$$u_3(t) \leqslant u_3(\xi_3) + \int_0^\omega |u_3'(t)|\,\mathrm{d}t < \ln\frac{A(a_{31}^M + a_{32}^M)}{\overline{a_{33}}} + c_3. \tag{2.23}$$

It follows from (2.4) and (2.18) that

$$\begin{aligned}\int_0^\omega a_{11}(t)\mathrm{e}^{u_1(t)}\,\mathrm{d}t &> \int_0^\omega (r_1(t) - D_1(t))\,\mathrm{d}t - \int_0^\omega a_{13}(t)\mathrm{e}^{u_3(t)}\,\mathrm{d}t \\ &\geqslant \omega\overline{(r_1 - D_1)} - a_{13}^M\frac{\omega A(a_{31}^M + a_{32}^M)}{a_{33}^L},\end{aligned}$$

which implies

$$u_1(\eta_1) > \ln \frac{\overline{(r_1 - D_1)} - a_{13}^M \dfrac{A(a_{31}^M + a_{32}^M)}{a_{33}^L}}{\overline{a_{11}}} \stackrel{\text{def}}{=} d_1. \tag{2.24}$$

This, together with (2.20), leads to

$$\begin{aligned} u_1(t) &\geqslant u_1(\eta_1) - \int_0^\omega |u_1'(t)|\,\mathrm{d}t \\ &> d_1 - c_1. \end{aligned} \tag{2.25}$$

It follows from (2.22) and (2.25) that

$$\max_{t\in[0,\omega]} |u_1(t)| < \max\{|\ln A| + c_1, |d_1| + c_1\} := R_1. \tag{2.26}$$

By a similar argument in (2.24) one can show that

$$u_2(\eta_2) > \ln \frac{\overline{(r_2 - D_2)} - a_{23}^M \dfrac{A(a_{31}^M + a_{32}^M)}{a_{33}^L}}{\overline{a_{22}}} \stackrel{\text{def}}{=} d_2. \tag{2.27}$$

It follows from (2.21) and (2.27) that

$$\begin{aligned} u_2(t) &\geqslant u_2(\eta_2) - \int_0^\omega |u_2'(t)|\,\mathrm{d}t \\ &> d_2 - c_2. \end{aligned} \tag{2.28}$$

This, together with (2.22), leads to

$$\max_{t\in[0,\omega]} |u_2(t)| < \max\{|\ln A| + c_2, |d_2| + c_2\} := R_2. \tag{2.29}$$

Noting that

$$\begin{aligned} \overline{(r_1 - D_1)}\omega &\leqslant a_{11}^M \int_0^\omega \mathrm{e}^{u_1(t)}\,\mathrm{d}t + a_{13}^M \int_0^\omega \mathrm{e}^{u_3(t)}\,\mathrm{d}t, \\ \overline{(r_2 - D_2)}\omega &\leqslant a_{22}^M \int_0^\omega \mathrm{e}^{u_2(t)}\,\mathrm{d}t + a_{23}^M \int_0^\omega \mathrm{e}^{u_3(t)}\,\mathrm{d}t, \end{aligned} \tag{2.30}$$

it follows from (2.6) and (2.17) that

$$\begin{aligned} \int_0^\omega a_{33}(t)\mathrm{e}^{u_3(t-\tau_2)}\,\mathrm{d}t &= \int_0^\omega a_{31}(t)\mathrm{e}^{u_1(t-\tau_1)}\,\mathrm{d}t + \int_0^\omega a_{32}(t)\mathrm{e}^{u_2(t-\tau_1)}\,\mathrm{d}t - \int_0^\omega r_3(t)\,\mathrm{d}t \\ &\geqslant a_{31}^L \int_0^\omega \mathrm{e}^{u_1(t-\tau_1)}\,\mathrm{d}t + a_{32}^L \int_0^\omega e^{u_2(t-\tau_1)}\,\mathrm{d}t - \overline{r_3}\omega \\ &= a_{31}^L \int_0^\omega e^{u_1(t)}\,\mathrm{d}t + a_{32}^L \int_0^\omega e^{u_2(t)}\,\mathrm{d}t - \overline{r_3}\omega \end{aligned}$$

$$\geqslant a_{31}^L \frac{\overline{(r_1 - D_1)}\omega - a_{13}^M \int_0^\omega e^{u_3(t)}\,dt}{a_{11}^M} + a_{32}^L \frac{\overline{(r_2 - D_2)}\omega - a_{23}^M \int_0^\omega e^{u_3(t)}\,dt}{a_{22}^M} - \overline{r_3}\omega, \tag{2.31}$$

which yields

$$u_3(\eta_3) \geqslant \ln \frac{a_{31}^L a_{22}^M \overline{(r_1 - D_1)} + a_{32}^L a_{11}^M \overline{(r_2 - D_2)} - \overline{r_3} a_{11}^M a_{22}^M}{a_{11}^M a_{22}^M a_{33}^M + a_{11}^M a_{32}^L a_{23}^M + a_{22}^M a_{31}^L a_{13}^M} \overset{\text{def}}{=} d_3. \tag{2.32}$$

It follows from (2.21) and (2.32) that

$$u_3(t) \geqslant u_3(\eta_3) - \int_0^\omega |u_3'(t)|\,dt > d_3 - c_3. \tag{2.33}$$

This, together with (2.23), leads to

$$\max_{t\in[0,\omega]} |u_3(t)| < \max \left\{ |\ln \frac{A(a_{31}^M + a_{32}^M)}{\overline{a_{33}}}| + c_3, |d_3| + c_3 \right\} := R_3. \tag{2.34}$$

Obviously, R_1, R_2 and R_3 in (2.26), (2.29) and (2.34) are independent of λ. Denote $M = R_1 + R_2 + R_3 + R_0$, where R_0 is taken sufficiently large such that each solution $(\alpha^*, \beta^*, \gamma^*)^T$ of the system of algebraic equations

$$\begin{aligned} &\overline{(r_1 - D_1)} - \overline{a_{11}} e^\alpha - \overline{a_{13}} e^\gamma + \overline{D_1} e^{\beta-\alpha} = 0,\\ &\overline{(r_2 - D_2)} - \overline{a_{22}} e^\beta - \overline{a_{23}} e^\gamma + \overline{D_2} e^{\alpha-\beta} = 0,\\ &-\overline{r_3} + \overline{a_{31}} e^\alpha + \overline{a_{32}} e^\beta - \overline{a_{33}} e^\gamma = 0, \end{aligned} \tag{2.35}$$

satisfies $\|(\alpha^*, \beta^*, \gamma^*)^T\| = |\alpha^*| + |\beta^*| + |\gamma^*| < M$ (if system (2.35) has at least one solution) and

$$B_1 + B_2 + B_3 < M,$$

where

$$B_1 = \max \left\{ |\ln A_1|, \left| \ln \frac{\overline{(r_1 - D_1)} - \overline{a_{13}} \dfrac{A_1(\overline{a_{31}} + \overline{a_{32}})}{\overline{a_{33}}}}{\overline{a_{11}}} \right| \right\},$$

$$B_2 = \max \left\{ |\ln A_1|, \left| \ln \frac{\overline{(r_2 - D_2)} - \overline{a_{23}} \dfrac{A_1(\overline{a_{31}} + \overline{a_{32}})}{\overline{a_{33}}}}{\overline{a_{22}}} \right| \right\},$$

$$B_3 = \max\left\{\left|\ln \frac{A_1(\overline{a_{31}}+\overline{a_{32}})}{\overline{a_{33}}}\right|,\right.$$
$$\left.\left|\ln \frac{\overline{a_{31}}\,\overline{a_{22}}\,\overline{(r_1-D_1)}+\overline{a_{32}}\,\overline{a_{11}}\,\overline{(r_2-D_2)}-\overline{r_3}\,\overline{a_{11}}\,\overline{a_{22}}}{\overline{a_{11}}\,\overline{a_{22}}\,\overline{a_{33}}+\overline{a_{11}}\,\overline{a_{32}}\,\overline{a_{23}}+\overline{a_{22}}\,\overline{a_{31}}\,\overline{a_{13}}}\right|\right\}, \tag{2.36}$$

in which

$$A_1 = \max\left\{\frac{\overline{r_1}}{\overline{a_{11}}}, \quad \frac{\overline{r_2}}{\overline{a_{22}}}\right\}. \tag{2.37}$$

We now set $\Omega = \{(u_1(t), u_2(t), u_3(t))^{\mathrm{T}} \in X : \|(u_1, u_2, u_3)^{\mathrm{T}}\| < M\}$. This satisfies the condition (a) in Lemma 2.1. When $(u_1(t), u_2(t), u_3(t))^{\mathrm{T}} \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap R^3$, $(u_1, u_2, u_3)^{\mathrm{T}}$ is a constant vector in R^3 with $|u_1| + |u_2| + |u_3| = M$. If system (2.35) has at least one solution, then we have

$$QN\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \overline{(r_1-D_1)} - \overline{a_{11}}\mathrm{e}^{u_1} - \overline{a_{13}}\mathrm{e}^{u_3} + \overline{D_1}\mathrm{e}^{u_2-u_1} \\ \overline{(r_2-D_2)} - \overline{a_{22}}\mathrm{e}^{u_2} - \overline{a_{23}}\mathrm{e}^{u_3} + \overline{D_2}\mathrm{e}^{u_1-u_2} \\ -\overline{r_3} + \overline{a_{31}}\mathrm{e}^{u_1} + \overline{a_{32}}\mathrm{e}^{u_2} - \overline{a_{33}}\mathrm{e}^{u_3} \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

If system (2.35) does not have a solution, we can directly derive

$$QN\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

This proves that condition (b) in Lemma 2.1 is satisfied.

In order to prove that condition (c) in Lemma 2.1 holds, we define $\phi : \mathrm{Dom}X \times [0,1] \to X$ by

$$\phi(u_1, u_2, u_3, \mu) = \begin{bmatrix} \overline{(r_1-D_1)} - \overline{a_{11}}\mathrm{e}^{u_1} \\ \overline{(r_2-D_2)} - \overline{a_{22}}\mathrm{e}^{u_2} \\ \overline{a_{31}}\mathrm{e}^{u_1} + \overline{a_{32}}\mathrm{e}^{u_2} - \overline{a_{33}}\mathrm{e}^{u_3} \end{bmatrix} + \mu \begin{bmatrix} -\overline{a_{13}}\mathrm{e}^{u_3} + \overline{D_1}\mathrm{e}^{u_2-u_1} \\ -\overline{a_{23}}\mathrm{e}^{u_3} + \overline{D_2}\mathrm{e}^{u_1-u_2} \\ -\overline{r_3} \end{bmatrix},$$

where $\mu \in [0,1]$ is a parameter. When $(u_1(t), u_2(t), u_3(t))^{\mathrm{T}} \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap R^3$, $(u_1, u_2, u_3)^{\mathrm{T}}$ is a constant vector in R^3 with $|u_1| + |u_2| + |u_3| = M$. We will show that when $(u_1, u_2, u_3)^{\mathrm{T}} \in \partial\Omega \cap \mathrm{Ker}L$, $\phi(u_1, u_2, u_3, \mu) \neq 0$. Assume the conclusion is not true, i.e., there is a constant vector $(u_1, u_2, u_3)^{\mathrm{T}}$ with $|u_1| + |u_2| + |u_3| = M$ satisfying $\phi(u_1, u_2, u_3, \mu) = 0$, i.e.,

$$\overline{(r_1-D_1)} - \overline{a_{11}}\mathrm{e}^{u_1} - \mu\overline{a_{13}}\mathrm{e}^{u_3} + \mu\overline{D_1}\mathrm{e}^{u_2-u_1} = 0,$$
$$\overline{(r_2-D_2)} - \overline{a_{22}}\mathrm{e}^{u_2} - \mu\overline{a_{23}}\mathrm{e}^{u_3} + \mu\overline{D_2}\mathrm{e}^{u_1-u_2} = 0,$$
$$\overline{a_{31}}\mathrm{e}^{u_1} + \overline{a_{32}}\mathrm{e}^{u_2} - \overline{a_{33}}\mathrm{e}^{u_3} - \mu\overline{r_3} = 0.$$

By similar arguments in (2.16), (2.19), (2.24), (2.27) and (2.32), we can derive

$$|u_i| < B_i, \quad i = 1, 2, 3,$$

where B_i is defined in (2.36). Thus, we have

$$|u_1| + |u_2| + |u_3| < M,$$

which leads to a contradiction. Using the property of topological degree and taking $J = I : \mathrm{Im}Q \to \mathrm{Ker}L, (u_1, u_2, u_3)^{\mathrm{T}} \to (u_1, u_2, u_3)^{\mathrm{T}}$, a standard and direct calculation shows that

$$\begin{aligned}
&\deg(JQN(u_1, u_2, u_3)^{\mathrm{T}}, \Omega \cap \mathrm{Ker}L, (0, 0, 0)^{\mathrm{T}}) \\
&\quad = \deg(\phi(u_1, u_2, u_3, 1), \Omega \cap \mathrm{Ker}L, (0, 0, 0)^{\mathrm{T}}) \\
&\quad = \deg(\phi(u_1, u_2, u_3, 0), \Omega \cap \mathrm{Ker}L, (0, 0, 0)^{\mathrm{T}}) \\
&\quad = \deg((\overline{(r_1 - D_1)} - \overline{a_{11}}\mathrm{e}^{u_1}, \overline{(r_2 - D_2)} - \overline{a_{22}}\mathrm{e}^{u_2}, \\
&\qquad \overline{a_{31}}\mathrm{e}^{u_1} + \overline{a_{32}}\mathrm{e}^{u_2} - \overline{a_{33}}\mathrm{e}^{u_3})^{\mathrm{T}}, \Omega \cap \mathrm{Ker}L, (0, 0, 0)^{\mathrm{T}}) \\
&\quad = -1.
\end{aligned}$$

By now we have proved that Ω satisfies all the requirements in Lemma 2.1. Hence, (2.2) has at least one ω-periodic solution. Accordingly, system (1.2) has at least one positive ω-periodic solution. This completes the proof.

3. Uniqueness and global stability

In this section, we formulate the uniqueness and global stability of the periodic solutions of system (1.2). It is immediate that if the positive periodic solution $z^*(t)$ is globally asymptotically stable then $z^*(t)$ is in fact unique. To this end, we first need to derive the uniform persistence of system (1.2).

Definition. System (1.2) is said to be uniform persistent if there exists a compact region $D \subset \mathrm{Int}\, R_+^3$ such that every solution $z(t)$ of (1.2) with initial conditions (1.3) eventually enters and remains in the region D.

Lemma 3.1. *Let $z(t) = (x_1(t), x_2(t), y(t))$ denote any positive solution of system (1.2) with initial conditions (1.3). Then there exists a $T_1 > 0$ such that*

$$0 < x_i(t) \leqslant M_i \ (i = 1, 2), \quad 0 < y(t) \leqslant M_3 \quad \text{for } t \geqslant T_1, \tag{3.1}$$

where

$$\begin{aligned}
&M_1 = M_2 > M_1^*, \\
&M_1^* = \max\left\{\frac{r_1^M}{a_{11}^L}, \frac{r_2^M}{a_{22}^L}\right\}, \quad M_3 = \frac{M_1(a_{31}^M + a_{32}^M)}{a_{33}^L}\mathrm{e}^{(a_{31}^M + a_{32}^M)M_1\tau_2}.
\end{aligned} \tag{3.2}$$

Lemma 3.2. *Let* (H1) *hold. Assume further that*

(H5) $(a_{31}^L + a_{32}^L)\min\left\{\dfrac{r_1^L - a_{13}^M M_3}{a_{11}^M}, \dfrac{r_2^L - a_{23}^M M_3}{a_{22}^M}\right\} > r_3^M,$

where M_3 is defined in (3.2). *Then system* (1.2) *is uniformly persistent, i.e., there exist $T > T_1$ and $m_i > 0 (i = 1, 2, 3)$ such that*

$$m_i < x_i(t) \leqslant M_i \quad (i = 1, 2), \quad m_3 < y(t) \leqslant M_3 \quad \text{for} \quad t \geqslant T, \tag{3.3}$$

where

$$\begin{aligned} m_i &= \frac{1}{2} m_i^*, \quad i = 1, 2, 3; \\ m_1^* &= m_2^* = \min\left\{\frac{r_1^L - a_{13}^M M_3}{a_{11}^M}, \frac{r_2^L - a_{23}^M M_3}{a_{22}^M}\right\}, \\ m_3^* &= \frac{m_1(a_{31}^L + a_{32}^L) - r_3^M}{a_{33}^M}. \end{aligned} \tag{3.4}$$

The proofs of Lemmas 3.1, 3.2 are standard and similar to those of Theorem 3.1, 3.2 of [38], we therefore omit them here.

Theorem 3.1. *In addition to* (H1)–(H5), *assume further that*

(H6) $\liminf\limits_{t \to \infty} A_i(t) > 0,$

where

$$\begin{aligned} A_1(t) =&\ a_{11}(t) - a_{31}(t + \tau_1) - \frac{D_2^M}{m_2} - a_{31}(t + \tau_1) M_3 \int_{t+\tau_1}^{t+\tau_1+\tau_2} a_{33}(s)\, ds; \\ A_2(t) =&\ a_{22}(t) - a_{32}(t + \tau_1) - \frac{D_1^M}{m_1} - a_{32}(t + \tau_1) M_3 \int_{t+\tau_1}^{t+\tau_1+\tau_2} a_{33}(s)\, ds; \\ A_3(t) =&\ a_{33}(t) - a_{13}(t) - a_{23}(t) \\ &- (r_3(t) + a_{31}(t) M_1 + a_{32}(t) M_2 + a_{33}(t) M_3) \int_t^{t+\tau_2} a_{33}(s)\, ds \\ &- a_{33}(t + \tau_2) M_3 \int_{t+\tau_2}^{t+2\tau_2} a_{33}(s)\, ds. \end{aligned} \tag{3.5}$$

Then (1.2)–(1.3) *has a unique positive ω-periodic solution $z^*(t) = (x_1^*(t), x_2^*(t), y^*(t))^{\mathrm{T}}$ which is globally asymptotically stable.*

Proof. Due to the conclusion of Theorem 2.1, we only need to show the global asymptotic stability of the positive periodic solutions of (1.2)–(1.3). Let $(x_1^*(t), x_2^*(t), y^*(t))^{\mathrm{T}}$ be a positive ω-periodic solution of system (1.2), and $(y_1(t), y_2(t), y_3(t))^{\mathrm{T}}$ be any positive solution of system (1.2). It follows from Lemma 3.2 that there exist positive constants $T>0$, M_i and m_i(defined by (3.2) and (3.4), respectively), such that for all $t \geqslant T$,

$$m_i < x_i^*(t) \leqslant M_i, i=1,2, \quad m_3 < y^*(t) \leqslant M_3,$$
$$m_i < y_i(t) \leqslant M_i, i=1,2,3. \tag{3.6}$$

We define

$$V_1(t) = |\ln x_1^*(t) - \ln y_1(t)| + |\ln x_2^*(t) - \ln y_2(t)|. \tag{3.7}$$

Calculating the Dini upper right derivative of $V_1(t)$ along the solution of (1.2), it follows for $t \geqslant T$ that

$$\begin{aligned} D^+V_1(t) &= \sum_{i=1}^{2} \left(\frac{\dot{x}_i^*(t)}{x_i^*(t)} - \frac{\dot{y}_i(t)}{y_i(t)} \right) \operatorname{sgn}(x_i^*(t) - y_i(t)) \\ &= \operatorname{sgn}(x_1^*(t) - y_1(t)) \Bigg\{ -a_{11}(t)(x_1^*(t) - y_1(t)) \\ &\quad - a_{13}(t)(y^*(t) - y_3(t)) + D_1(t) \left(\frac{x_2^*(t)}{x_1^*(t)} - \frac{y_2(t)}{y_1(t)} \right) \Bigg\} \\ &\quad + \operatorname{sgn}(x_2^*(t) - y_2(t)) \Bigg\{ -a_{22}(t)(x_2^*(t) - y_2(t)) \\ &\quad - a_{23}(t)(y^*(t) - y_3(t)) + D_2(t) \left(\frac{x_1^*(t)}{x_2^*(t)} - \frac{y_1(t)}{y_2(t)} \right) \Bigg\} \\ &\leqslant -a_{11}(t)|x_1^*(t) - y_1(t)| + (a_{13}(t) + a_{23}(t))|y^*(t) - y_3(t)| \\ &\quad - a_{22}(t)|x_2^*(t) - y_2(t)| + \tilde{D}_1(t) + \tilde{D}_2(t), \end{aligned} \tag{3.8}$$

where

$$\tilde{D}_1(t) = \begin{cases} D_1(t) \left(\dfrac{x_2^*(t)}{x_1^*(t)} - \dfrac{y_2(t)}{y_1(t)} \right), & x_1^*(t) > y_1(t), \\ D_1(t) \left(\dfrac{y_2(t)}{y_1(t)} - \dfrac{x_2^*(t)}{x_1^*(t)} \right), & x_1^*(t) < y_1(t), \end{cases}$$

$$\tilde{D}_2(t) = \begin{cases} D_2(t) \left(\dfrac{x_1^*(t)}{x_2^*(t)} - \dfrac{y_1(t)}{y_2(t)} \right), & x_2^*(t) > y_2(t), \\ D_2(t) \left(\dfrac{y_1(t)}{y_2(t)} - \dfrac{x_1^*(t)}{x_2^*(t)} \right), & x_2^*(t) < y_2(t). \end{cases}$$

We estimate $\tilde{D}_1(t)$ under the following three cases:

(i) If $x_1^*(t) > y_1(t)$, then

$$\tilde{D}_1(t) \leqslant \frac{D_1(t)}{x_1^*(t)}(x_2^*(t) - y_2(t)) \leqslant \frac{D_1^M}{m_1}|x_2^*(t) - y_2(t)|;$$

(ii) If $x_1^*(t) < y_1(t)$ then

$$\tilde{D}_1(t) \leqslant \frac{D_1(t)}{y_1(t)}(y_2(t) - x_2^*(t)) \leqslant \frac{D_1^M}{m_1}|x_2^*(t) - y_2(t)|;$$

(iii) If $x_1^*(t) = y_1(t)$, by a similar argument one can show that the same conclusion as (i) and (ii) holds.

Combining the conclusions in (i)–(iii), we obtain

$$\tilde{D}_1(t) \leqslant \frac{D_1^M}{m_1}|x_2^*(t) - y_2(t)|. \tag{3.9}$$

By a similar argument in the above discission one can show that

$$\tilde{D}_2(t) \leqslant \frac{D_2^M}{m_2}|x_1^*(t) - y_1(t)|. \tag{3.10}$$

It follows from (3.8), (3.9) and (3.10) that

$$\begin{aligned} D^+V_1(t) \leqslant& -a_{11}(t)|x_1^*(t) - y_1(t)| - a_{22}(t)|x_2^*(t) - y_2(t)| \\ &+ (a_{13}(t) + a_{23}(t))|y^*(t) - y_3(t)| \\ &+ \frac{D_1^M}{m_1}|x_2^*(t) - y_2(t)| + \frac{D_2^M}{m_2}|x_1^*(t) - y_1(t)|. \end{aligned} \tag{3.11}$$

Define

$$V_{21}(t) = |\ln y^*(t) - \ln y_3(t)|. \tag{3.12}$$

Calculating the Dini upper right derivative of $V_{21}(t)$ along the solution of (1.2), we derive for $t \geqslant T$ that

$$\begin{aligned} D^+V_{21}(t) =& \left(\frac{\dot{y}^*(t)}{y^*(t)} - \frac{\dot{y}_3(t)}{y_3(t)}\right) \operatorname{sgn}(y^*(t) - y_3(t)) \\ =& \operatorname{sgn}(y^*(t) - y_3(t)) \{-a_{33}(t)(y^*(t-\tau_2) - y_3(t-\tau_2)) \\ &+ a_{31}(t)(x_1^*(t-\tau_1) - y_1(t-\tau_1)) \\ &+ a_{32}(t)(x_2^*(t-\tau_1) - y_2(t-\tau_1))\} \\ =& \operatorname{sgn}(y^*(t) - y_3(t)) \left\{-a_{33}(t)(y^*(t) - y_3(t)) \right. \\ &+ a_{31}(t)(x_1^*(t-\tau_1) - y_1(t-\tau_1)) \\ &+ a_{32}(t)(x_2^*(t-\tau_1) - y_2(t-\tau_1)) \\ &\left. + a_{33}(t)\int_{t-\tau_2}^{t} (\dot{y}^*(u) - \dot{y}_3(u))\,\mathrm{d}u \right\} \\ \leqslant& -a_{33}(t)|y^*(t) - y_3(t)| \\ &+ a_{31}(t)|x_1^*(t-\tau_1) - y_1(t-\tau_1)| \\ &+ a_{32}(t)|x_2^*(t-\tau_1) - y_2(t-\tau_1)| \\ &+ a_{33}(t)\left|\int_{t-\tau_2}^{t} (\dot{y}^*(u) - \dot{y}_3(u))\,\mathrm{d}u\right|. \end{aligned} \tag{3.13}$$

On substituting (1.2) into (3.13), we obtain

$$\begin{aligned} D^+V_{21}(t) \leqslant & -a_{33}(t)|y^*(t)-y_3(t)| \\ & + a_{31}(t)|x_1^*(t-\tau_1)-y_1(t-\tau_1)| \\ & + a_{32}(t)|x_2^*(t-\tau_1)-y_2(t-\tau_1)| \\ & + a_{33}(t)\left|\int_{t-\tau_2}^{t}\Big[(-r_3(u)+a_{31}(u)y_1(u-\tau_1)\right. \\ & + a_{32}(u)y_2(u-\tau_1)-a_{33}(u)y_3(u-\tau_2))(y^*(u)-y_3(u)) \\ & + a_{31}(u)y^*(u)(x_1^*(u-\tau_1)-y_1(u-\tau_1)) \\ & + a_{32}(u)y^*(u)(x_2^*(u-\tau_1)-y_2(u-\tau_1)) \\ & \left. - \ a_{33}(u)y^*(u)(y^*(u-\tau_2)-y_3(u-\tau_2))\Big] \, \mathrm{d}u\right|. \end{aligned} \tag{3.14}$$

Noting the fact in (3.6), we derive from (3.14) that for $t \geqslant T+\tau$

$$\begin{aligned} D^+V_{21}(t) \leqslant & -a_{33}(t)|y^*(t)-y_3(t)| \\ & + a_{31}(t)|x_1^*(t-\tau_1)-y_1(t-\tau_1)| \\ & + a_{32}(t)|x_2^*(t-\tau_1)-y_2(t-\tau_1)| \\ & + a_{33}(t)\int_{t-\tau_2}^{t}\Big[(r_3(u)+a_{31}(u)M_1 \\ & + a_{32}(u)M_2+a_{33}(u)M_3)|y^*(u)-y_3(u)| \\ & + a_{31}(u)M_3|x_1^*(u-\tau_1)-y_1(u-\tau_1)| \\ & + a_{32}(u)M_3|x_2^*(u-\tau_1)-y_2(u-\tau_1)| \\ & + \ a_{33}(u)M_3|y^*(u-\tau_2)-y_3(u-\tau_2)|\Big] \, \mathrm{d}u. \end{aligned} \tag{3.15}$$

Define

$$\begin{aligned} V_{22}(t) = & \int_{t-\tau_1}^{t} a_{31}(s+\tau_1)|x_1^*(s)-y_1(s)| \, \mathrm{d}s \\ & \int_{t-\tau_1}^{t} a_{32}(s+\tau_1)|x_2^*(s)-y_2(s)| \, \mathrm{d}s \\ & + \int_{t}^{t+\tau_2}\int_{s-\tau_2}^{t} a_{33}(s)\Big\{(r_3(u)+a_{31}(u)M_1 \\ & + a_{32}(u)M_2+a_{33}(u)M_3)|y^*(u)-y_3(u)| \end{aligned}$$

$$
\begin{aligned}
&+ a_{31}(u)M_3|x_1^*(u-\tau_1) - y_1(u-\tau_1)| \\
&+ a_{32}(u)M_3|x_2^*(u-\tau_1) - y_2(u-\tau_1)| \\
&\left.+ a_{33}(u)M_3|y^*(u-\tau_2) - y_3(u-\tau_2)|\right\} \mathrm{d}u\,\mathrm{d}s. \qquad (3.16)
\end{aligned}
$$

It follows from (3.15) and (3.16) that for $t \geqslant T + \tau$

$$
\begin{aligned}
D^+V_{21}(t) + \dot{V}_{22}(t) \leqslant\ & -a_{33}(t)|y^*(t) - y_3(t)| \\
&+ a_{31}(t+\tau_1)|x_1^*(t) - y_1(t)| \\
&+ a_{32}(t+\tau_1)|x_2^*(t) - y_2(t)| \\
&+ \int_t^{t+\tau_2} a_{33}(s)\,\mathrm{d}s \left\{ (r_3(t) + a_{31}(t)M_1 \right. \\
&+ a_{32}(t)M_2 + a_{33}(t)M_3)|y^*(t) - y_3(t)| \\
&+ a_{31}(t)M_3|x_1^*(t-\tau_1) - y_1(t-\tau_1)| \\
&+ a_{32}(t)M_3|x_2^*(t-\tau_1) - y_2(t-\tau_1)| \\
&\left. + a_{33}(t)M_3|y^*(t-\tau_2) - y_3(t-\tau_2)| \right\}. \qquad (3.17)
\end{aligned}
$$

We define

$$
V_2(t) = V_{21}(t) + V_{22}(t) + V_{23}(t), \qquad (3.18)
$$

in which

$$
\begin{aligned}
V_{23}(t) = M_3 & \int_{t-\tau_1}^{t} \int_{l+\tau_1}^{l+\tau_1+\tau_2} a_{33}(s)a_{31}(l+\tau_1)|x_1^*(l) - y_1(l)|\,\mathrm{d}s\,\mathrm{d}l \\
&+ M_3 \int_{t-\tau_1}^{t} \int_{l+\tau_1}^{l+\tau_1+\tau_2} a_{33}(s)a_{32}(l+\tau_1)|x_2^*(l) - y_2(l)|\,\mathrm{d}s\,\mathrm{d}l \\
&+ M_3 \int_{t-\tau_2}^{t} \int_{l+\tau_2}^{l+2\tau_2} a_{33}(s)a_{33}(l+\tau_2)|y^*(l) - y_3(l)|\,\mathrm{d}s\,\mathrm{d}l. \qquad (3.19)
\end{aligned}
$$

It then follows from (3.17), (3.18), and (3.19) that for $t \geqslant T + \tau$

$$
\begin{aligned}
D^+V_2(t) \leqslant\ & -a_{33}(t)|y^*(t) - y_3(t)| + a_{31}(t+\tau_1)|x_1^*(t) - y_1(t)| \\
&+ a_{32}(t+\tau_1)|x_2^*(t) - y_2(t)| \\
&+ \int_t^{t+\tau_2} a_{33}(s)\,\mathrm{d}s \left\{ (r_3(t) + a_{31}(t)M_1 \right.
\end{aligned}
$$

$$+ a_{32}(t)M_2 + a_{33}(t)M_3)|y^*(t) - y_3(t)|$$
$$+ a_{31}(t+\tau_1)M_3 \int_{t+\tau_1}^{t+\tau_1+\tau_2} a_{33}(s)\,\mathrm{d}s|x_1^*(t) - y_1(t)|$$
$$+ a_{32}(t+\tau_1)M_3 \int_{t+\tau_1}^{t+\tau_1+\tau_2} a_{33}(s)\,\mathrm{d}s|x_2^*(t) - y_2(t)|$$
$$\left. + a_{33}(t+\tau_2)M_3 \int_{t+\tau_2}^{t+2\tau_2} a_{33}(s)\,\mathrm{d}s|y^*(t) - y_3(t)| \right\}. \tag{3.20}$$

We now define a Lyapunov functional $V(t)$ as

$$V(t) = V_1(t) + V_2(t). \tag{3.21}$$

Then it follows from (3.11), (3.20), and (3.21) that for $t \geqslant T + \tau$

$$D^+V(t) \leqslant -\sum_{i=1}^{2} A_i(t)|x_i^*(t) - y_i(t)| - A_3(t)|y^*(t) - y_3(t)|, \tag{3.22}$$

where $A_1(t), A_2(t)$ and $A_3(t)$ are defined in (3.5).

By hypothesis (H6), there exist constants $\alpha_i > 0\,(i = 1,2,3)$ and $T^* \geqslant T + \tau$, such that

$$A_i(t) \geqslant \alpha_i > 0 \quad \text{for } t \geqslant T^*. \tag{3.23}$$

Integrating both sides of (3.22) on interval $[T^*, t]$,

$$V(t) + \sum_{i=1}^{2} \int_{T^*}^{t} A_i(s)|x_i^*(s) - y_i(s)|\,\mathrm{d}s + \int_{T^*}^{t} A_3(s)|y^*(s) - y_3(s)|\,\mathrm{d}s \leqslant V(T^*). \tag{3.24}$$

It follows from (3.23) and (3.24) that

$$V(t) + \sum_{i=1}^{2} \alpha_i \int_{T^*}^{t} |x_i^*(s) - y_i(s)|\,\mathrm{d}s + \alpha_3 \int_{T^*}^{t} |y^*(s) - y_3(s)|\,\mathrm{d}s \leqslant V(T^*) \quad \text{for } t \geqslant T^*. \tag{3.25}$$

Therefore, $V(t)$ is bounded on $[T^*, \infty)$ and also

$$\begin{aligned} &\int_{T^*}^{\infty} |x_i^*(s) - y_i(s)|\,\mathrm{d}s < \infty, \ i = 1,2, \\ &\int_{T^*}^{\infty} |y^*(s) - y_3(s)|\,\mathrm{d}s < \infty. \end{aligned} \tag{3.26}$$

By Lemma 3.1, $|x_i^*(t) - y_i(t)|(i = 1,2)$ and $|y^*(t) - y_3(t)|$ are bounded on $[T^*, \infty)$.

On the other hand, it is easy to see that $\dot{x}_i^*(t)(i=1,2)$, $\dot{y}^*(t)$ and $\dot{y}_i(t)(i=1,2,3)$ are bounded for $t \geqslant T^*$. Therefore, $|x_i^*(t) - y_i(t)|(i=1,2)$ and $|y^*(t) - y_3(t)|$ are uniformly continuous on $[T^*, \infty)$. By Barbalat's Lemma (see lemma 1.2.2, 1.2.3 in Gopalsamy [16]), we can conclude that

$$\lim_{t\to\infty} |x_i^*(t) - y_i(t)| = 0, \quad i = 1,2;$$
$$\lim_{t\to\infty} |y^*(t) - y_3(t)| = 0.$$

This completes the proof.

Finally, we give three examples to illustrate the feasibility of our main results.

Example 1. We consider the following Lotka–Volterra predator-prey model with prey dispersal and time delays

$$\dot{x}_1(t) = x_1(t)[15 + \sin t - 20x_1(t) - 8y(t)] + 0.5(x_2(t) - x_1(t)),$$
$$\dot{x}_2(t) = x_2(t)[15 - \sin t - 18x_2(t) - 7y(t)] + 0.3(x_1(t) - x_2(t)),$$
$$\dot{y}(t) = y(t)\left[-0.1 + 0.01\sin t + 5x_1\left(t - \frac{1}{10}\right) + 6x_2\left(t - \frac{1}{10}\right) - (18 + \sin(t))y\left(t - \frac{1}{10^3}\right)\right]. \quad (3.27)$$

It is easy to examine that the coefficients of system (3.27) satisfy all assumptions in Theorems 2.1, 3.1. Thus, by Theorem 3.1, system (3.27) has a unique positive 2π-periodic solution which is globally asymptotically stable. Numerical integration of system (3.27) can now be carried out using standard algorithms. We used the **dde23** package in MATLAB. Numerical simulation shows that system (3.27) has a unique positive 2π-periodic solution which is globally asymptotically stable (see Fig. 1).

Example 2. We consider another delayed periodic Lotka–Volterra diffusive predator-prey system

$$\dot{x}_1(t) = x_1(t)[5 + \sin t - 3x_1(t) - 6y(t)] + 6(x_2(t) - x_1(t)),$$
$$\dot{x}_2(t) = x_2(t)[4 - \sin t - 2x_2(t) - 5y(t)] + 5(x_1(t) - x_2(t)),$$
$$\dot{y}(t) = y(t)\left[-2 + (4 + \sin t)x_1\left(t - \frac{1}{10}\right) + 3x_2(t - \frac{1}{10}) - 2y\left(t - \frac{1}{10^3}\right)\right]. \quad (3.28)$$

It is easy to show that $\overline{(r_1 - D_1)} = -1$, $\overline{(r_2 - D_2)} = -1$. Obviously, (H2) and (H3) don't hold for system (3.28). However, numerical simulation shows that system (3.28) still has at least one positive 2π-periodic solution (see Fig. 2).

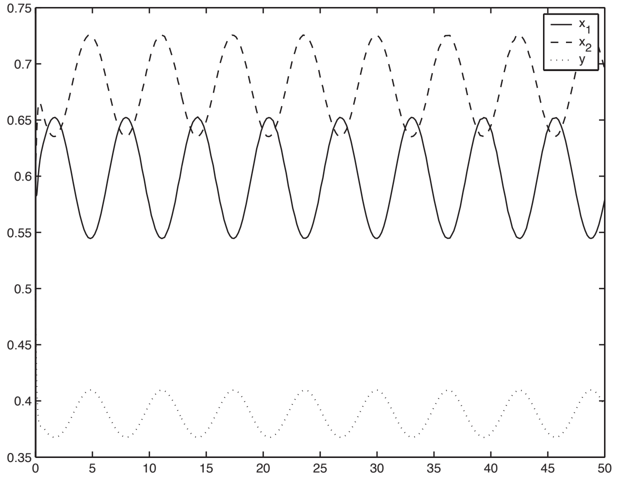

Fig. 1. The existence of positive 2π-periodic solution of system (3.27).

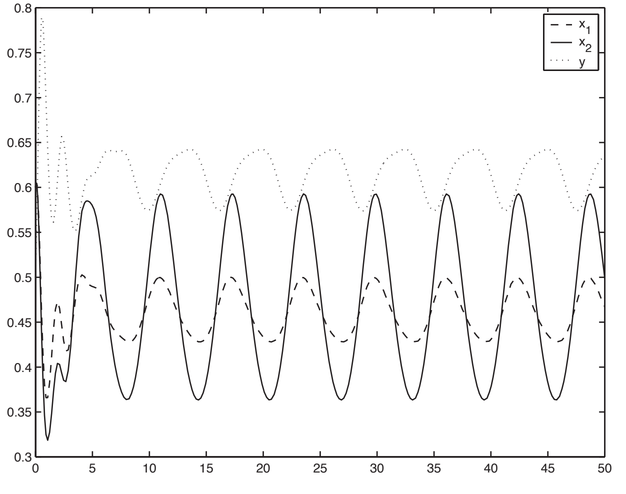

Fig. 2. The positive 2π-periodic solution of system (3.28).

Example 3. We consider the following delayed periodic Lotka–Volterra predator-prey system with prey dispersal

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)[5 + \sin t - 5x_1(t) - 6y(t)] + 0.5(x_2(t) - x_1(t)),\\
\dot{x}_2(t) &= x_2(t)[4 - \sin t - 5x_2(t) - 5y(t)] + 0.3(x_1(t) - x_2(t)),\\
\dot{y}(t) &= y(t)\left[-3 + (2 + \sin t)x_1\left(t - \frac{1}{10}\right) + 2x_2\left(t - \frac{1}{10}\right) - 2y\left(t - \frac{1}{10^3}\right)\right].
\end{aligned} \tag{3.29}$$

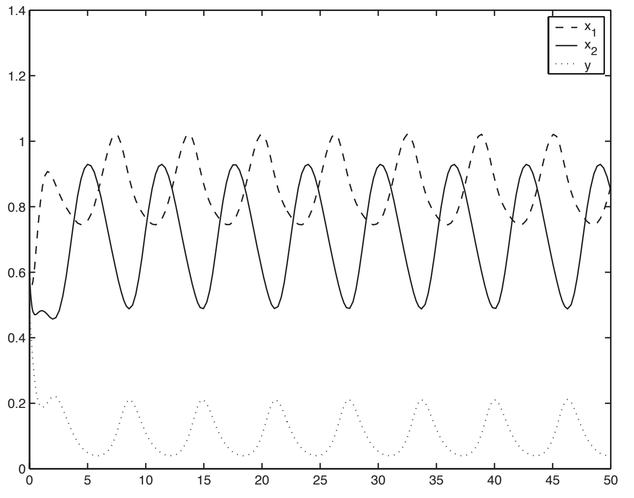

Fig. 3. The positive 2π-periodic solution of system (3.29).

It is easy to show that, the coefficients in system (3.29) don't satisfy (H5). However, numerical simulation shows that system (3.29) still has at least one positive 2π-periodic solution (see Fig. 3).

Remark. In this paper, based on the model proposed by Kuang and Takeuchi in [21], we discussed a nonautonomous two-species Lotka–Volterra predator-prey system with prey dispersal and time delays due to gestation and negative feedback of the predator. By using Gaines and Mawhin's continuation theorem of coincidence degree theory and by means of a suitable Lyapunov functional, a set of easily verified sufficient conditions have been established to guarantee the existence, uniqueness and global stability of positive periodic solutions. Numerical simulation has been presented to show that system (1.2) will have a unique positive periodic solution if its coefficients satisfy (H1)–(H6). We would like to mention here that, by Examples 2, 3 we see that the conditions (H1)–(H5) are not necessary for the existence of positive periodic solutions to system (1.2). This shows that our results for the existence of positive periodic solutions and persistence of the system have room for improvement. The results on the uniqueness of positive periodic solutions may also have room to be improved. However, significant improvement seems to be difficult. The next step in the study on predator-prey model with dispersion would be to incorporate time delays to the self-regulated terms of the prey in both patches. We leave these for our future work.

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