MAGNETIC CONNECTIVITY OF CORONAL FIELDS: GEOMETRICAL VERSUS TOPOLOGICAL DESCRIPTION

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ABSTRACT

We analyse the mapping produced by the field lines which connect photospheric areas of positive and negative magnetic polarity on the Sun. The geometrical quantities independent of the direction of such a mapping (from positive to negative polarity, and vice versa) are introduced. They yield a complete description of the field line connectivity in coronal magnetic configurations and, in particular, in quasi-separatrix layers (QSLs). The elementary magnetic flux tubes are enormously squashed in QSLs without giving the true jump in the field-line mapping, as it occurs at the genuine separatrix surfaces. So the QSLs may be present even in topologically trivial magnetic configurations. This is illustrated by an example relevant to solar flare events.

INTRODUCTION: QSL VERSUS SEPARATRIX SURFACE

Investigations of coronal magnetic fields extrapolated from photospheric magnetograms show a systematic spatial correlation between the locations of energy release in solar flares and the regions of strong variation of the field line connectivity (Démolulin *et al.*, 1997). Such regions, called quasi-separatrix layers (QSLs), are thought to be the plausible places for the magnetic reconnection process (Priest and Démoulin, 1995).

The most of magnetic field lines in solar active regions connect domains of positive and negative polarity of the photospheric plane, say, $z = 0$. Let the location of their footpoints in this plane be represented depending on the polarity by the radius-vector $\mathbf{r}_+ = (x_+, y_+)$ or $\mathbf{r}_- = (x_-, y_-)$. The connections of the footpoints by the field lines determine two mutually inverse mappings ${}_{+}\Pi_{-} : \mathbf{r}_+ \mapsto \mathbf{r}_-$ and ${}_{-}\Pi_{+} : \mathbf{r}_- \mapsto \mathbf{r}_+$. We shall simply use Π if we refer to aspects valid for both mappings. Also the functional forms $(x_-(\mathbf{r}_+), y_-(\mathbf{r}_+)) \equiv {}_{+}\Pi_{-}(\mathbf{r}_+)$ and $(x_+(\mathbf{r}_-), y_+(\mathbf{r}_-)) \equiv {}_{-}\Pi_{+}(\mathbf{r}_-)$ will be used further for the mappings.

The mapping Π is discontinuous at the footpoints of field lines threading magnetic nulls in the corona or touching the photosphere, since the magnetic flux tubes enclosing such field lines split at the nulls or at the touching points (Seehafer, 1986; Titov *et al.*, 1993). The corresponding discontinuities serve as indicators for the separatrix field lines and surfaces, which are genuine topological features responsible for the current sheet formation in evolving coronal magnetic fields. It is worth to emphasise that the coordinates $(x_\pm, y_\pm)$ in this case need not to be Cartesian because the discontinuities are revealed in any system of coordinates irrespective of the metric.

However, with the help of the metric or Cartesian coordinates one can determine not only the genuine separatrices but also the QSLs. The integrity of the flux tubes is preserved within the QSLs and so the mapping Π remains continuous at the corresponding footpoints, but the shape of their cross-sections strongly change along the flux tubes. Thus, instead of the true discontinuities in Π at the intersection of the genuine separatrices with the photosphere, there are continuous but rapid variations in Π at the photospheric cross-sections of QSLs. These variations can only be detected by using the metric, which enables us to measure and compare the distances between the footpoints in one polarity and corresponding footpoints in the other polarity. In this respect QSLs and separatrices are qualitatively different objects. Indeed, ignoring the above metrical information about Π and using a proper continuous change of coordinates, it is possible to eliminate the rapid variations in Π and thereby the QSLs themselves, while the discontinuities of Π and hence the corresponding separatrices are not removable in this way. As we can see below the coronal field may be topologically trivial, or, in other words, diffeomorphic to a simple arcade-like field, but its geometrical structure is rather complicated due to the presence of QSL.

In spite of this mathematical difference, the QSLs may physically be as important as genuine separatrix surfaces. This is supported for instance by the following arguments. In most of the coronal volume the quasi-static conditions are fulfilled, so that the magnetic field evolves through a sequence of force-free equilibria. Such conditions, however, may easily break down in QSLs, where due to a strong variation of the field line connectivity the rearrangement of the field lines during the evolution of the configuration may occur faster than in other places. This in turn implies a locally large acceleration of plasma and hence a locally unbalanced and enhanced Lorentz force with corresponding concentrations of the current density. So ultimately the inertia of plasma may cause the formation of strong current layers in QSLs. In this respect the properties of QSLs and genuine separatrix surfaces are similar to each other.

The proper measure for QSLs has been found in our previous paper (Titov *et al.*, 1999). Here we concentrate on the full geometrical description of magnetic connectivity by emphasising the difference between geometrical and topological properties of magnetic fields.

GEOMETRICAL DESCRIPTION OF THE MAGNETIC CONNECTIVITY

The mapping Π is locally described by its differential $\mathrm{d}\Pi$, which is a linear mapping from the plane tangent to the photosphere at one footpoint to a similar plane at the other footpoint. Assume hereafter that $(x_\pm, y_\pm)$ are measured in one Cartesian system of coordinates covering the whole photospheric plane, then the differential $\mathrm{d}\underset{+-}{\Pi}$ is represented by Jacobian matrix

$$\underset{+-}{\mathcal{D}} = \begin{pmatrix} \frac{\partial x_-}{\partial x_+} & \frac{\partial x_-}{\partial y_+} \\ \frac{\partial y_-}{\partial x_+} & \frac{\partial y_-}{\partial y_+} \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \tag{1}$$

Following to Titov *et al.* (1999) $\underset{+-}{\mathcal{D}}$ can be decomposed in the non-degenerate case as

$$\underset{+-}{\mathcal{D}} = \mathcal{R}_{\gamma_-}\, \Lambda\, \mathcal{R}_{\gamma_+}^{-1}, \tag{2}$$

where $\mathcal{R}_{\gamma_+}$ and $\mathcal{R}_{\gamma_-}$ are matrices of rotations on some angles γ_+ and γ_-, respectively, and

$$\Lambda = \mathrm{diag}(\lambda_1, \lambda_2) \tag{3}$$

is the matrix of anisotropic dilation.

If one introduces the complex values $(\mathrm{i} = \sqrt{-1})$

$$\xi = a + d + \mathrm{i}\,(b - c), \tag{4}$$
$$\zeta = a - d + \mathrm{i}\,(b + c), \tag{5}$$

then their arguments determine the above angles as

$$\gamma_+ = (\arg\xi + \arg\zeta)/2, \tag{6}$$
$$\gamma_- = (\arg\zeta - \arg\xi)/2, \tag{7}$$

while their moduli yield the dilation coefficients as

$$\lambda_1 = (|\xi| + |\zeta|)/2, \tag{8}$$
$$\lambda_2 = (|\xi| - |\zeta|)/2. \tag{9}$$

Eq. (2) reveals the local geometrical properties of magnetic connectivity. Indeed, consider the whole magnetic field in the corona as a collection of elementary flux tubes (EFTs) the cross-section of which scale $\sim \delta^2$ $(\to 0)$. Let for each of these tubes the photospheric cross-section in the positive polarity be the rectangle $(\lambda_1^{-1/2}\delta) \times (|\lambda_2|^{-1/2}\delta)$ whose first side is oriented at the angle γ_+. Then Eq. (2) means that the cross-section of such a tube in the negative polarity is also a rectangle but of the size $(\lambda_1^{1/2}\delta) \times (|\lambda_2|^{1/2}\delta)$ with the first side inclined at the angle γ_-. One can see from here that for both these rectangles the aspect ratio is the same and equal to $(\lambda_1/|\lambda_2|)^{1/2}$, where (see Titov *et al.*, 1999)

$$\lambda_1/|\lambda_2| = Q/2 + \sqrt{Q^2/4 - 1}, \tag{10}$$
$$Q = (a^2 + b^2 + c^2 + d^2)/|ad - bc|. \tag{11}$$

By construction $\lambda_1/|\lambda_2|$, and so Q, is invariant with respect to the direction of Π. At $Q \gg 1$ the quantity $Q \approx \lambda_1/|\lambda_2|$ defines the degree of squashing of EFTs in QSLs and can be used as one of the four independent quantities describing the magnetic connectivity. Another quantity independent of Q and invariant (up to the sign) with respect to Π is expressed in the positive polarity as

$$K \equiv \log|\lambda_1\lambda_2| = \log|ad - bc| \tag{12}$$

and simply mapped by $\underset{+-}{\Pi}$ onto the negative polarity by changing its sign on the opposite. This quantity determines the degree of expansion $(K > 0)$ or contraction $(K < 0)$ of the cross-sections in EFTs. Together with the fields of

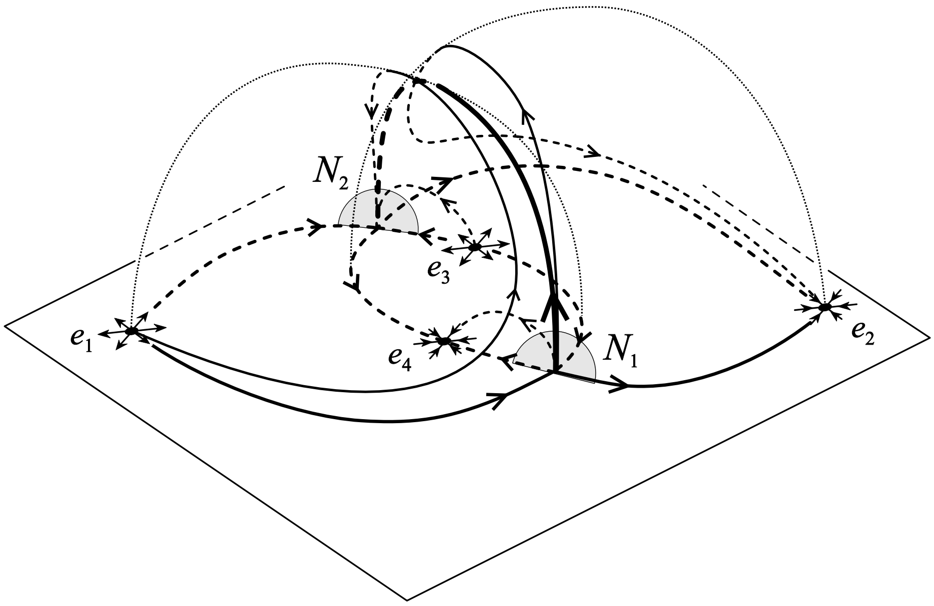

Fig. 1. The separator in the configuration with four point-like sources is a field line (the thickest solid line) connecting two nulls N_1 and N_2 (cf. Figure 10 in (Sweet, 1969)). The light grey semicircles represent the fan surfaces near these nulls.

directions determined by the angles γ_+ and γ_- the quantities Q and K yield a complete geometrical description of the field-line connectivity in the corona.

ILLUSTRATIVE EXAMPLE

Let us see what the above theory yields for the configuration formed by two bipolar groups of sunspots. Such a configuration is considered for a long time (cf. Sweet (1969) and Gorbachev and Somov (1988)) as a basic one for two-ribbon solar flares and is quite instructive for our purposes. In an idealised form the corresponding sunspots are represented by point-like sources $e_1, \ldots, e_4$ of potential field, whose topological structure is non-trivial due to the presence of null points in the field. For a wide range of positions and strengths of the sources the separatrix surfaces of the configuration are formed by the field lines emanating from the two null points N_1 and N_2 as shown in Figure 1. At the intersection of two separatrix surfaces there is a special field line called 'separator', which is considered as a preferable place for the development of current sheet and magnetic reconnection process in the configuration.

To model a more realistic configuration with the distributed photospheric magnetic field, Gorbachev and Somov (1988) modified the configuration by placing the sources below the photosphere (Figure 2) and identified the magnetic topology of the idealised and modified configurations (Figure 3). This trick enabled them to introduce the separator in the configuration but at the cost of a rather strong assumption on the topology of the field below the photosphere. On this basis they managed to explain several important observational features of the two-ribbon flares.

In fact, the geometry of coronal field in this configuration is rich enough to obtain the same results without any additional assumption on the sub-photospheric magnetic structure. If one computes, for example, the distribution of Q for the case shown in Figure 2, then it becomes clear that such a configuration contains the QSL in which the maximums of $Q \sim 10^6$ (see Figure 4a). The region of the maximum in each polarity correspond to the photospheric cross-section of this QSL, which we will call further QSL-trace. It is a narrow strip passing along the inversion line nearby one of the footpoints of the separator for the corresponding idealised source model (cf. Figures 3 and 4). So in the light of the presumable relation of QSLs with the magnetic reconnection the QSL-traces can be identified with the H_α-ribbons of solar flares – the fact which Gorbachev and Somov (1988) explained by a hyperbolic geometry of the field lines around the separator. From our approach, however, it is clear that for the coronal configurations with the distributed photospheric field only this geometry itself has a real meaning, while the indicated separator is no more than an artifact originated by the auxiliary sub-photospheric point sources. On the other hand, the trick with the change of the sunspots by the corresponding point sources may be useful for an approximate estimation of the coronal magnetic structure.

The distribution of $|\Delta| = \lambda_1|\lambda_2| \equiv 10^K$ in logarithmic scale (Figure 4b) also reveals the QSL in the configuration under study – the QSL-traces in this case are narrow strips of $|\Delta| \sim 1$ which separate two areas of small $|\Delta|$ with the minimums $\sim 10^{-2}$ (extended light grey regions) and end up in the smaller areas of large $|\Delta|$ with the maximums

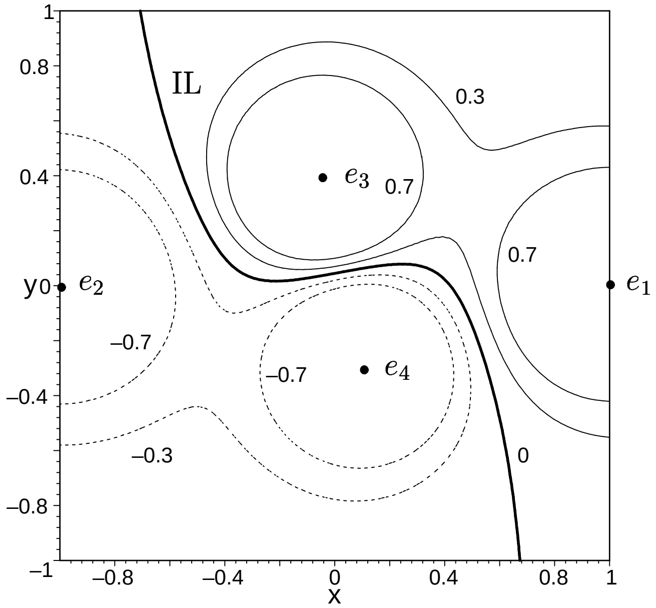

Fig. 2. The photospheric distribution of the normal magnetic field for the point sources $e_1 = -e_2 = 0.6$ and $e_3 = -e_4 = 0.4$ placed below the photosphere ($z = 0$) on the plane $z = -0.1$ (cf. Figure 1c in (Gorbachev and Somov, 1988)). The thick solid line is the inversion line (IL).

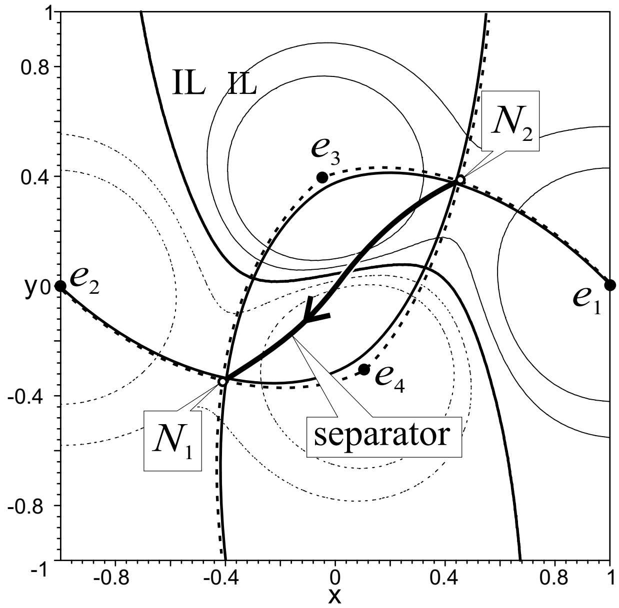

Fig. 3. The separator in the model of Gorbachev and Somov (1988). The dashed lines passing through the fictive sources and nulls N_1 and N_2 are intersections of the fictive separatrix surfaces with the plane of the sources, while the neighbouring solid lines are the respective intersections with the photosphere.

$\sim 10^2$ (compact dark grey regions). Since $|\Delta|$ has mutually inverse values at the ends of a given field line, the areas of minimums and maximums of $|\Delta|$ must be connected to each other. Thus the $|\Delta|$-distribution enables us to identify two pairs of small and large magnetic flux tubes whose projection on the photosphere are traced in Figure 4b by dots and pluses. These tubes contract towards the ends of the QSL-traces, which suggest a natural explanation of the bright 'knots' at the flare ribbon ends. The energy released inside the QSL must be channelled to the photosphere by the surrounding field lines, so that their convergent character at the ends of the QSL-traces leads to concentration of the released energy in these regions and so to a formation of such bright 'knots' (cf. Gorbachev and Somov, 1988). In spite of the connectivity being continuous in the whole field, the flux tubes appear to be qualitatively distinct structural elements, which in the evolving configuration must interact with each other through the QSL. The character of such an interaction can be appraised for the particular evolution in which the ideal plasma flow only "shuffles" the field lines in configuration without temporal variation of the field itself. One can see from Figure 4b that such an evolution would have the following peculiarity: if a field line passes from one of the above mentioned tubes to another, so that one of its footpoint crosses slowly the narrow QSL-trace, then its other footpoint will sweep along the QSL-trace in the other polarity. Figures 2 and 4b also show that the QSL-traces connect the modelled sunspots, so that they serve as channels through which the field lines have an opportunity to switch from one sunspot to another, both of the same polarity. The aspect ratio of the QSL-traces in the considered example is of the order of $\sqrt{Q} \approx 10^3$. A crossing of the QSL-trace by a field line with a footpoint velocity $v \sim 1$ km/s would require for its sweeping at the other end $v \sim 10^3$ km/s – a value comparable with the Alfvén speed in the solar active regions. This demonstrates that the violation of the quasi-static conditions for such regions is reached foremost in QSLs, which in turn implies a current accumulation there and possible magnetic reconnection as well. The limiting case of the point sources lying in the photospheric plane (Figure 1) corresponds in the considered model to a traditional reconnection along the separator field line when the QSL degenerates into the genuine separatrix surface passing through the corresponding nulls and point sources.

CONCLUSIONS

The geometrical and topological properties of magnetic connectivity, that is those which are based on the metric or are independent of it, are often confused in astrophysics. This is a source of misleading concepts and results rather than only a question of terminology. In our approach the borderline between the geometrical and topological descriptions is clearly defined. We have shown that the complete description of the magnetic connectivity is given by the four metrical quantities which characterise squashing and contraction-expansion of the elementary magnetic flux tubes. The singularities and discontinuities in the photospheric distributions of these quantities correspond to the topological features of the coronal field, while the topologically trivial regions are characterised by smooth distributions of these values. The considered example of the topologically simple configuration with two bipolar groups of sunspots

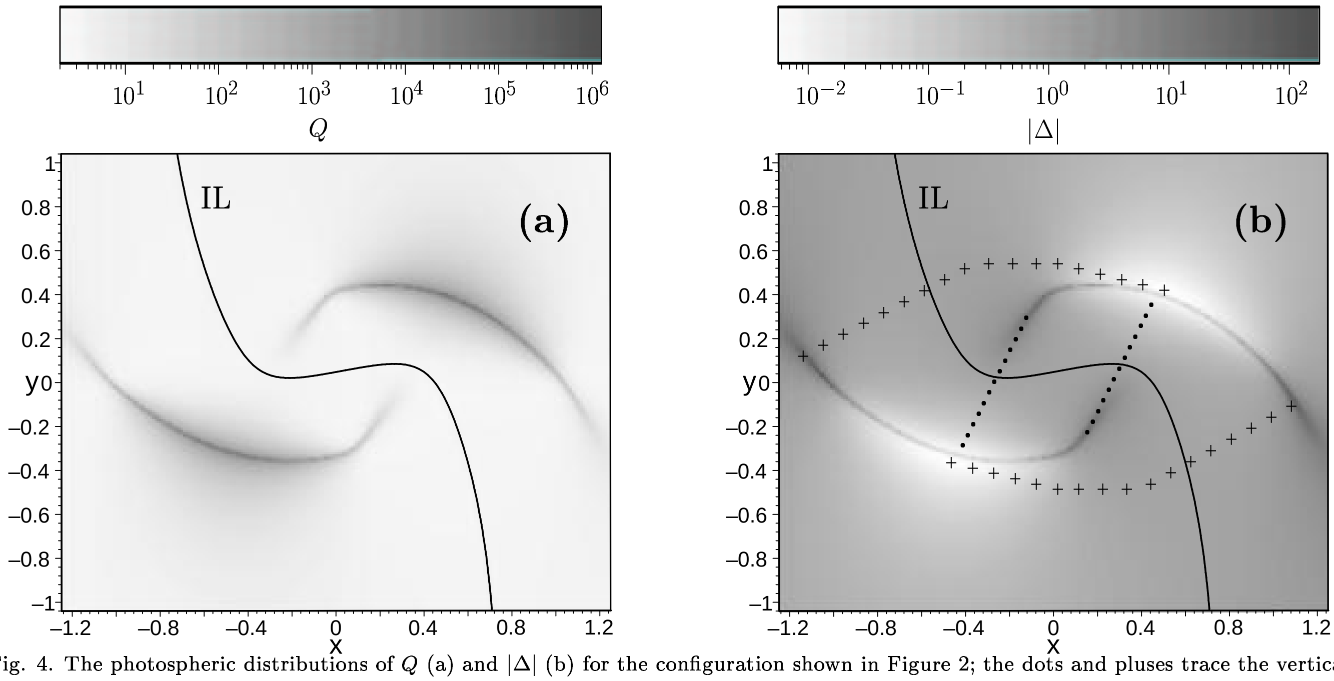

Fig. 4. The photospheric distributions of Q (a) and $|\Delta|$ (b) for the configuration shown in Figure 2; the dots and pluses trace the vertical projection of the four interacting flux tubes on the photosphere.

has illustrated this approach 'in action' by demonstrating its high efficiency in analysing the structure of coronal magnetic fields and especially one of their geometrical features – the quasi-separatrix layers.

ACKNOWLEDGEMENTS

We gratefully acknowledge financial support from the Volkswagen-Foundation.

REFERENCES

Démoulin, P., L. G. Bagalá, C. H. Mandrini, J. C. Hénoux and M. G. Rovira, Quasi-separatrix layers in solar flares. II. Observed magnetic configurations, *Astron. Astrophys.*, **325**, 305–317, 1997.

Gorbachev, V. S. and B. V. Somov, Photospheric vortex flows as a cause for two-ribbon flares: a topological model, *Solar Phys.*, **117**, 77-88, 1988.

Priest, E. R. and P. Démoulin, Three-dimensional magnetic reconnection without null points 1. Basic theory of magnetic flipping, *J. Geophys. Res.*, **100**, 23443–23463, 1995.

Seehafer, N., On the magnetic field line topology in solar active regions, *Solar Phys.*, **105**, 223–235, 1986.

Sweet, P. A., Mechanisms of solar flares, *ARA&A*, **7**, 149–177, 1969.

Titov, V. S., E. R. Priest and P. Démoulin, Conditions for the appearance of "bald patches" at the solar surface, *Astron. Astrophys.*, **276**, 564–570, 1993.

Titov, V. S., P. Démoulin and G. Hornig, Quasi-separatrix layers: refined theory and its application to solar flares, in Magnetic fields and solar processes, ed. A. Wilson, 9-th Europ. Meeting on Sol. Phys., **ESA SP-448**, pp. 715–722, ESA Public. Div. ESTEC, Noordwijk, The Netherlands, 1999.