

# Mathematics of Fluids and Plasmas 1+2 - Outline

February 9, 2007

# Mathematics of Fluids and Plasmas 1: Fluid Dynamics

1. Fundamentals (5 lectures)
  - Vector fields, flows and flux.
  - Vector potential.
  - Representation of fields: fieldlines/streamlines, contours, flux surfaces
  - Vector identities, Gauss, Stokes theorem
2. Conservation laws (3 lectures)
  - Conservation of momentum, pressure tensor, Navier-Stokes Eq., Euler Eq.
  - Conservation of mass, energy, equation of state.
3. Common approximations (2 lectures)
  - Incompressible, irrotational, potential flow
  - Bernoulli's theorem
4. Laplace's equation (3 lectures)
  - Laplace's equation, boundary conditions, uniqueness theorem
  - Solution in different coordinate systems, Expansion in spherical harmonics
  - Green's function, Helmholtz decomposition
5. Waves (2)
  - Sound waves
  - Linearisation, dispersion relations, wave properties
6. Vorticity (3)
  - Vorticity and circulation (definitions)
  - Kelvin's circulation theorem
  - Vorticity evolution equation (derive)
  - Vortex tubes
7. Viscous flow (3)
  - Stress tensor, viscous stresses, viscosity
  - Irreversibility of viscous flow (energy dissipation)
  - Flow through a pipe, Reynolds number, high and low Re flow
8. Turbulence and Chaos (qualitatively) (1)

## Mathematics of Fluids and Plasmas 2: MHD and the Sun

1. Electromagnetism [4 lecture]
  - Maxwell's eq.s
  - Electrostatics,
  - Magnetostatic fields, magnetic effect of currents
  - Electrodynamics, Waves
2. Introduction to properties of plasmas, especially on the sun [1 lecture].
3. Equations of Magnetohydrodynamics (MHD) [5 lectures]
  - Lorentz force, MHD equations, importance of terms,
  - Diffusion and frozen in flux,
  - Magnetic field lines and flux tubes.
4. MHD solutions [4 lectures]
  - Hydrostatic pressure balance, plasma beta,
  - Potential fields,
  - Force-free fields, coronal arcades,
  - Grad-Shafranov equation
5. Waves [3 lectures]
  - Linearised MHD equations,
  - Sound waves, Alfvén waves, magnetoacoustic waves
6. Solar applications: [5 lectures]
  - Magnetic reconnection,
  - Magnetic helicity
  - Dynamo
  - Solar flares
  - CME's