RETINAL VESSEL SEGMENTATION USING A FINITE ELEMENT BASED BINARY LEVEL SET METHOD

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Abstract. In this paper we combine a few techniques to label blood vessels in the matched filter (MF) response image by using a finite element based binary level set method. An operator-splitting method is applied to numerically solve the Euler-Lagrange equation from minimizing an energy functional. Unlike the traditional MF methods, where a threshold is difficult to be selected, our method can automatically get more precise blood vessel segmentation using an enhanced edge information. In order to demonstrate the good performance, we compare our method with a few other methods when they are applied to a publicly available standard database of coloured images (with manual segmentations available too).

1. Introduction

The main task of the medical image analysis is the extraction of an appropriate feature of the image data including organs, tissues, lesions, tumours, exudates, blood vessels etc. Assessment of the characteristics of blood vessels plays an important role in medical diagnoses. A variety of medical imaging techniques such as ultrasound, X-ray imaging, magnetic resonance imaging (MR), computed tomography (CT), and fluorescein angiography are capable of obtaining data on blood vessels. In the majority of images noise and lack of contrast pose significant challenges to the extraction of blood vessels. A number of methods have been...
proposed for the segmentation of vascular structures (see [1] for a review). Extracting blood vessel of medical images is even more challenging due to its large number of vessels and some of them are tiny and vague. Existing vessel segmentation methods include: rule-based methods and comprises vessel tracking [2] [3], matched filter responses [2] [4] [5] [6], multithreshold probing in [8], topology adaptive snakes [9], morphology-based techniques [10] [11] [12], neural network approaches [13], pattern recognition and multiscale approaches [14] [15] [16] [17] [18], tight-frame methods [20] [21], deformable model and level set approaches [26] [27] [28].

Among the various vessel segmentation methods, the matched filter [4] (MF) is widely used for its advantages of simplicity and effectiveness. It convolves the image with multiple matched filters for the extraction of objects of interest. Considering the fact that the cross-section of a vessel can be modeled as a Gaussian function, a series of Gaussian-shaped filter can be used to match the vessels for detection. Because MF method does not yield a satisfactory classification using a single global threshold, Hoover et al. [5] present a vessel segmentation method that uses local and region-based properties at each pixel. Based on the fact that wavelet kernels are especially suitable for detecting edges in signals such as blood vessel borders, Soares et al. [15] adopt Gabor wavelets as they provide directional selectivity and fine tuning to specific frequencies, enabling noise reduction. Cai et al. [20] apply the framelet-based approach to denoise and smooth the possible boundary and sharpen the region of tube-like structures for identifying blood vessels in real 2D/3D images. Due to a central light reflection of some large vessels, Wang et al. [19] adopt a 2-D Hermite function intensity model and Lupascu et al. [14] build a 41-D feature vector for each pixel, encoding information on the local intensity structure, spatial properties, and geometry at multiple scales. However, most of the above proposed methods are far from being fully automated since they require user interaction for selecting appropriate thresholds.

One way to overcome this difficulty is the curve evolution method for vessel segmentation, where no threshold need be preselected. Kass et al. [22] introduced the active contour model or snake. A multitude of powerful deformable models for medical image segmentation have been proposed [23]. Frangi et al. [24] propose to reconstruct 2-D vessel boundaries or 3-D vessel walls using deformable surface models represented by B-spline surfaces. However, it is not possible to employ parameterized geometric models to effectively deal with whole vessel trees, as the models would be required to change topology during evolution. The level set method and in particular the motion by mean curvature proposed by Osher and Sethian [25] have been used extensively, because it allows for cusps, corners, and automatic topological change. In the area of vessel segmentation, Lorigo et al. [26] use a geodesic active contour model based on the level set method for segmentation of brain vasculatures, and the abdominal aorta from high-contrast MRA and CT images. Chen et al. [27] present a hybrid approach to accurate quantification of vascular structures from magnetic resonance angiography (MRA) images using level set methods and deformable geometric models constructed with 3-D Delaunay triangulation. Zonoobi et al. [28] propose a scheme called the gradient compensated geodesic active contours, which compensates for low gradients near edges of thin vessels and is implemented based on level set, to detect vasculatures in both synthetic volumetric image and real 3D MRA images. Dong et al. [21] combine ideas of wavelet tight frames based image restoration with ideas of the total variation based piecewise constant Mumford-Shah model proposed by [29] to capture key features.
of vessel-like structures. However, most of above level set methods rely on the edge function \[31\] (depending on the image gradient) to stop the curve evolution and the curve may pass through the boundary if the image is very noisy.

In this paper, we label blood vessels in the MF response image by using a binary level set method based on Mumford-Shah model (see \[29\]), which can detect contours with both sharp edges and blurred edges (see \[32\][38]), instead of thresholding method.

As one variant of some recently proposed piecewise constant level set methods (\[32\][33][34][35]), the binary level set method is closely related to the phase field model (\[36\][37]). The image is segmented through Euler-Lagrange equation obtained via the calculus of variations to minimize a Mumford-Shah energy functional under some constraints. In this paper, for better efficiency of implements, we use a finite element and an operator-splitting method to solve the Euler-Lagrange equation for minimization of the energy functional. The finite-element method (FEM) solution strategy is implemented because of the following benefits or convenience: 1) The FEM approach is based on a variational formulation which is naturally obtained from the energy functional minimization and relatively easy to analyze through functional analysis framework; 2) The FEM approach provides an analytical representation of the solution through its basis functions; 3) The FEM approach can easily deal with Neumann type of boundary conditions (or called natural boundary conditions in FEM context). This type of boundary conditions is used in this paper and frequently used in various image processing problems. A description of operator-splitting methods can be found in \[39\][40][41][43][44]. Although each individual technique mentioned above has been used before, a method which combined all these techniques seems to be new to detect or segment images with special structures (i.e. vascular structure or blood vessels). More importantly, examples shown in Section 5 demonstrate that the combined method performs very well in blood vessel segmentation.

The outline of the paper is as follows: In Section 2, the pixel MF response image generation is described using Gaussian filter. Section 3 presents segmentation model using binary level set based on Mumford-Shah model. An operator-splitting scheme is explained in Section 4. Results and discussions are in Section 5. Finally, conclusions are made in Section 6.

2. The Matched Filter

When the RGB coloured images are visualized separately, the green channel shows the best vessel background contrast whereas the red and blue channels show low contrast and very noisy\[4\]. Therefore, the green channel image will be selected as the gray-level image to be processed by our method. As pointed out in \[4\], the gray-level profile of the cross section of a blood vessel can be approximated by a Gaussian shaped curve. The concept of matched filter detection is used to detect piecewise linear segments of blood vessels in retinal images. Blood vessels usually have poor local contrast. The two-dimensional matched filter kernel is designed to convolve with the original image $Img$ in order to enhance the blood vessels. A prototype Gaussian matched filter kernel is expressed as

$$ker(x, y, a, b) = -exp\left(-\frac{(a^{-1}(x - b))^2}{2\sigma^2}\right), |y| \leq L/2$$
where $a$ is the dilation parameter, $b$ is the displacement scalar, and $L$ is the length of the segmentation for which the vessel is assumed to have a fixed orientation. The direction of the vessel is assumed to be aligned along the $y$-axis. Because a vessel may be oriented at any directions, the kernel needs to be rotated at any possible angles and the maximum convolution attains when the angle is aligned with the direction the vessel.

For each position and considered scaling value, we are interested in the maximum of the convolution of the $2$-D image ($Img(x, y)$) and the $2$-D Gaussian kernel ($ker(x, y, a, b)$) over all possible orientations because it indicates the vessel at that orientation. Mathematically this can be expressed as

$$M_{ker}(a, b) = \max_\theta (r_\theta (ker(x, y, a, b)) \ast Img(x, y))$$

(1)

where $r_\theta$ represents a rotation of the Gaussian kernel at an angle $\theta$ and $\ast$ denotes convolution. Figure 1 (b) shows the response of the MF to a Gaussian function for an original image Figure 1 (a). We can see that there are mainly blood vessel region left in the image enhanced by the matched filter.

3. The Binary Level Set Method

The response of MF usually includes unwanted information and a preselected threshold has to be used for segmentation. We thus use the binary level set method combined with the Mumford-Shah functional for the segmentation (see [29]). It turns out that the result is better when we apply the method to the MF response image (see our numerical tests later). The notation and description of the method follows [32]. Let us assume that the blood vessel region is enclosing $\Omega_b \subset$ the region $\Omega$ of the MF response image. For the binary level set method, we use a discontinuous level set function $\phi$ as

$$\phi(x) = \begin{cases} 1 & \text{if } x \in \text{int}(\Omega_b), \\ -1 & \text{if } x \in \text{ext}(\Omega_b). \end{cases}$$

(2)

A piecewise constant approximation $u$ for a given image $u_0$ is constructed as the sum

$$u(\phi, \bar{c}) = \frac{c_1}{2} (\phi + 1) - \frac{c_2}{2} (\phi - 1).$$

(3)

where $u = c_1$ in $\Omega_b$ and $u = c_2$ outside $\Omega_b$.

Then, we minimize the following Mumford-Shah functional to find a segmentation of a given image $u_0$ using the above binary level set expression:

$$F(\phi, \bar{c}) = \frac{1}{2} \int_\Omega |u(\phi, \bar{c}) - u_0|^2 dx + \beta \int_\Omega |\nabla \phi| dx$$

(4)

where $\phi^2 = 1$, $\bar{c} = \{c_1, c_2\}$, and $u(\phi, \bar{c})$ is defined in (3).

Considering the constraints imposed on the level set function $\phi^2 = 1$, the segmentation problem is to solve the following constrained minimization problem

$$\min_{\phi, \bar{c}} F(\phi, \bar{c}), \text{ subject to } \phi^2 = 1.$$
The penalty formulation of problem (5) is:

\[ \min_{\phi, \vec{c}} F_\eta(\phi, \vec{c}) \]

where

\[ F_\eta(\phi, \vec{c}) = \frac{1}{2} \int_\Omega |u(\phi, \vec{c}) - u_0|^2 dx + \beta \int_\Omega |\nabla \phi| dx + \frac{1}{4\eta} \int_\Omega (\phi^2 - 1)^2 dx \]

(7)

In order to find a minimizer for (7), we shall find \( \vec{c} \) and \( \phi \) that satisfy \( \frac{\partial F_\eta}{\partial \vec{c}} = 0 \) and \( \frac{\partial F_\eta}{\partial \phi} = 0 \). \( F_\eta \) is quadratic with respect to \( \vec{c} \). For a given \( \phi^n \), the minimizer of \( F_\eta \) satisfies

\[ \sum_{j=1}^{2} \int_\Omega \psi_i(\phi^n) \psi_j(\phi^n) c_{ij} = \int_\Omega u_0 \psi_i(\phi^n), \quad i = 1, 2, \]

(8)

where

\[ \psi_1 = \frac{1}{2}(\phi + 1), \]
\[ \psi_2 = -\frac{1}{2}(\phi - 1). \]

For a fixed \( \vec{c} \), the steepest decent method or gradient flow in \( \phi \) for energy functional (7) gives the following equation for the level set function \( \phi \):

\[ \phi_t = \beta \nabla \cdot \left( \frac{\nabla \phi}{|\nabla \phi|} \right) - (u(\phi, \vec{c}) - u_0) \frac{\partial u}{\partial \phi} - \frac{1}{\eta} (|\phi|^2 - 1) \phi \]

(10)

with boundary condition

\[ \frac{\nabla \phi}{|\nabla \phi|} \cdot \vec{n} = 0 \quad \text{on} \quad \partial \Omega \]

From (10) and (11), time-evolution variational formulations of (7) read

\[ \begin{cases} \int_\Omega \phi_t v dx = \int_\Omega -\beta \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \cdot \nabla v dx - \int_\Omega (u(\phi, \vec{c}) - u_0) \frac{\partial u}{\partial \phi} v dx \\ -\frac{1}{\eta} \int_\Omega (|\phi|^2 - 1) \phi v dx \end{cases} \]

for all \( v \) in an appropriate space \( V \). We will use a finite element method for the spatial approximation of problem (10). Let \( h \) be a spatial discretization step. A finite element triangulation of \( \Omega \) being denoted as \( T_h \), we approximate the functional space \( V \) by the finite element spaces

\[ V_h = \{ \mathbf{v}_h \in C^0(\Omega), \quad \mathbf{v}_h|_T \in P_1 \times P_1, \quad \forall T \in T_h \} \]

where \( P_1 \) is the space of the polynomials in the variables of degree \( \leq 1 \).

4. THE MARCHUK-YANENKO OPERATOR-SPLITTING SCHEME

An efficient way to solve (10) or (12) is the Marchuk-Yanenko Operator-splitting Scheme (see [40]). That is
• For $n \geq 0$, knowing the current step $\phi_{\text{current}}^n$, we compute $\phi_{\text{current}}^{n+\frac{1}{2}} \in V_h$ via the solution of

$$
\int_{\Omega} \frac{\phi_{\text{current}}^{n+\frac{1}{2}} - \phi_{\text{current}}^n}{\Delta t} v dx = \int_{\Omega} -\beta \left( \frac{\nabla \phi_{\text{current}}^{n+\frac{1}{2}}}{|\nabla \phi_{\text{current}}^{n+\frac{1}{2}}|} \right) \cdot \nabla v dx - \int_{\Omega} (u(\phi_{\text{current}}^{n+\frac{1}{2}}, \vec{c}) - u_0) \frac{\partial u(\phi_{\text{current}}^{n+\frac{1}{2}}, \vec{c})}{\partial \phi} \cdot \nabla v dx
$$

(14)

where $v \in V_h$, $\Delta t$ is the time discretization step and $\phi_{\text{current}}^n$ is the solution at $t = n \Delta t$.

• Given $\phi_{\text{current}}^{n+\frac{1}{2}}$, compute $\phi_{\text{next}}^{n+1} \in V_h$ from the penalty term:

$$
\int_{\Omega} \frac{\phi_{\text{next}}^{n+1} - \phi_{\text{current}}^{n+\frac{1}{2}}}{\Delta t} v dx = -\frac{1}{\eta} \int_{\Omega} ((|\phi_{\text{next}}^{n+1}|^2 - 1) \phi_{\text{next}}^{n+1} v dx
$$

(15)

where $\forall v \in V_h$.

• Given $\phi_{\text{next}}^{n+1}$ compute the next step $\phi_{\text{next}}^{n+1}$ by the following equation

$$
\phi_{\text{next}}^{n+1} = \mathcal{P}(\phi_{\text{next}}^{n+1})
$$

(16)

where

$$
\mathcal{P}(x) = \begin{cases} 
1 & \text{if } x \geq 0 \\
-1 & \text{if } x \leq 0.
\end{cases}
$$

(17)

The solution of the nonlinear problem (14) is solved by the following Picard or fixed point iteration:

$$
\int_{\Omega} \frac{\phi_{\text{new}} - \phi_{\text{current}}^n}{\Delta t} v dx = \int_{\Omega} -\beta \left( \frac{\nabla \phi_{\text{new}}}{|\nabla \phi_{\text{old}}|} \right) \cdot \nabla v dx - \int_{\Omega} (u(\phi_{\text{old}}, \vec{c}) - u_0) \frac{\partial u(\phi_{\text{old}}, \vec{c})}{\partial \phi} \cdot \nabla v dx.
$$

(18)

Some analysis of the convergence of such a method may be found in [42]. In [42] we showed the iterative convergence in the $L^2$ norm for the full equation (12) with a linear Laplacian if $\Delta t/\eta < 1$. In the current paper we deal with the $\eta$ related term separately (due to splitting). So we expect that the method would converge if $\Delta t$ is sufficiently small. It seems that all our numerical examples demonstrate this point. Even though it is not convergent, we can still use Newton’s iterative method. Normally we start with initial value $\phi_{\text{old}} = \phi_{\text{current}}$. A CG method can be used to get a $\phi_{\text{new}}$ through the above linear equation (see [44]). We use this $\phi_{\text{new}}$ as $\phi_{\text{old}}$ again and get another $\phi_{\text{new}}$, and so on.

The solution of problem (15) can be seen weakly as a solution of

$$
\phi_{\text{next}}^{n+1} + \alpha((|\phi_{\text{next}}^{n+1}|^2 - 1) \phi_{\text{next}}^{n+1} = \phi_{\text{next}}^{n+\frac{1}{2}}
$$

(19)

where $\alpha = \Delta t/\eta$, since (15) is true for all $v$. We can write

$$
\phi_{\text{penalty}}^{n+1} = \frac{\phi_{\text{next}}^{n+\frac{1}{2}}}{1 - \alpha + \alpha |\phi_{\text{penalty}}^{n+1}|^2}
$$

(20)

where $|\phi_{\text{penalty}}^{n+1}|$ satisfies the following equation if $\alpha \leq 1$:

$$
\alpha z^3 + (1 - \alpha)z - |\phi_{\text{penalty}}^{n+1}| = 0
$$

(21)
This is a cubic equation and algebraic formulas are available for its closed form solutions. The discriminant of the cubic equation is \( \Delta = -4\alpha(1-\alpha)^3 - 27\alpha^2|\phi^{n+\frac{1}{2}}|^2 \).

The theory of cubic equation indicates that when \( \Delta < 0 \) the cubic equation has one real root and two non-real complex conjugate root. We choose \( \Delta t \) and \( \eta \) such that \( \alpha \leq 1 \), then \( \Delta < 0 \) and the equation (21) has a unique real solution. Equation (21) can be solved by the Newton’s method with \( |\phi^{n+\frac{1}{2}}| \) as its initial guess if \( \alpha < 1 \).

When \( \alpha = 1 \) or \( \eta = \Delta t \) the solution of (21) can be easily obtained and we have

\[
\phi^{n+1}_{penalty} = \frac{\phi^{n+\frac{1}{2}}}{|\phi^{n+\frac{1}{2}}|^{2/3}} \quad \text{if} \quad \phi^{n+\frac{1}{2}} \neq 0
\]

From the above discussion we see that the operator-splitting method is very efficient to deal with the constraint \( \phi^2 = 1 \) of the binary level set method for the retinal vessel segmentation.

Finally, the principal steps of the algorithm for our retinal segmentation are:

**Algorithm 1:**
- Initial \( \phi_0^{current} \) by \( \phi_0 \), \( n = 0 \).
- Compute \( c_1(\phi_0^{current}) \) and \( c_2(\phi_0^{current}) \) by (8).
- Solve PDE problem using (14), (15), and (16) to obtain \( \phi^{n+1}_{next} \).
- Check wether the solution is stationary. If not, \( n = n + 1 \) and repeat.

## 5. Results and Discussions

To enable comparative assessment, we use images and associated manual segmentations from two public data sets available on the web, DRIVE [16] and STARE [5] with the initial parameters \( \Delta t = 0.08, \eta = 0.1, \beta = 5 \), and

\[
\phi(x, y) = \begin{cases} 
1 & \text{if} \quad -\sqrt{(x-350)^2 + (x-304)^2 + 100} \geq 0 \\
-1 & \text{if} \quad -\sqrt{(x-350)^2 + (x-304)^2 + 100} \leq 0
\end{cases}
\]
in Algorithm 1. Both DRIVE and STARE includes ground truth segmentations for their images. The field of view (FOV) is approximately 50 deg for images of both sets. The DRIVE data set consists of 40 images (768 \( \times \) 584, 8 bits per channel). The images are in compressed JPEG format and have been divided into a training and a test set, each containing 20 images. They were manually segmented by three observers trained by an ophthalmologist. From STARE we used 20 images (700 \( \times \) 605 pixels, 8 bits per channel). The FOV in the images are approximately 650 \( \times \) 550 pixels in diameter. Ten of the images contain pathology. Two observers segmented all images. The segmentations of the two observers are fairly different, in that the second observer segmented much more of the thinner vessels than the first one; we evaluate performance against the more demanding segmentations by the first observer. As stated before, we work in the green channel.

We use the same \( a \) and \( L \) (\( a = 6 \) and \( L = 9 \)) in [4] as the kernel of MF.

\[
M_{ker}(x, y; a, b)
\]
in equation (1) is calculated with a number of discrete values of \( \theta \) from 0° up to 170° with an increment of 10°. Illustrative segmentation results for an image 0162 from STARE [5] database to explain our method are shown in Figure 1. Figure 1 shows the response of the MF to a Gaussian function for the image 0162. We can see clearly that after the application of MF to enhance blood vessels in the original image Figure 1(a), the blood vessels with precise edge are reserved in Figure 1(b), which is suitable to use our binary level set segmentation method. Figure 1(c)-(m) show retinal segmentation results at different \( n \) in Algorithm 1. Figure 1(m) shows the segmentation result when the solution is stationary.
MF simultaneously enhances some small blob-like and short linear structures in the process of matching the blood vessels in retinal images, so there are some unavoidable small blobs and short lines to be detected in final segmentation result (Figure 1 (m)). Considering the fact that the non-vessel structures always disconnect with the blood vessel network, the morphological open operator is applied for removing blob-like structures and some short linear structures in the resulting binary image after retinal vessel segmentation. The open operators with linear structural elements are applied to the binary image along twelve directions ($0^\circ, 15^\circ, 30^\circ, \cdots, 165^\circ$). The length of linear structure elements is set as 15 pixels.

In Figure 2, we show the final segmentation result for image 0162 from the STARE database with $\beta = 5$ and compare our result with result and hand-labeled ground truth images reported in Hoover et al. [5] (which belongs to the MF method). We can see clearly that the retinal vessel segmentation result contains more small vessels and is closer to the hand-labeled ground truth.

Figure 3 shows the final segmentation results for image 0162 from the STARE database with $\beta = 0.5$ and $\beta = 50$. We know that small $\beta$ corresponds to little noise removal and large $\beta$ yields an image with few small vessels. $\beta = 5$ is a good choice for all the vessel segmentation we have done so far.

Figure 4 shows another example using image 01 from the DRIVE database [16]. We use the SE against SP of the true/false positive ($TP/FP$) and true/false negative ($TN/FN$) fractions [45] to compare our method against some recent retinal segmentation methods. The performance measure of the segmentation methods were obtained from their original paper or their websites.

As usual, we define

\[
\text{Specificity} (SP) = \frac{TN}{FP + TN},
\]

\[
\text{Sensitivity} (SE) = \frac{TP}{TP + FN}.
\]

The accuracy ($ACC$) for one image is the fraction of pixels correctly classified

\[
ACC = \frac{TP + TN}{P + N} = \frac{TP + TN}{TP + FN + FP + TN}.
\]

We compute also the kappa values (a measure for observer agreement, where the two observers are the gold standard and the segmentation method) [14]

\[
kappa = \frac{P(A) - P(E)}{1 - P(E)},
\]

where $P(A) = (TP + TN)/(P + N)$ is the proportion of times the two observers agree, while $P(E) = (TP + FP)/(P + N) \ast (TP + FN)/(P + N) + (1 - (TP + FP)/(P + N))(1 - (TP + FN)/(P + N))$ is the proportion of time the two observers are expected to agree by chance alone.

Table 1 shows the sensitivity and specificity results of our method in comparison with the method in Hoover et al. [5] for 20 images in the STARE databases. Table 2 shows the sensitivity and specificity results of our method in comparison with the method in Sraal et al. [16] for 20 images in the DRIVE databases. The results demonstrate that, on STARE and DRIVE database, our proposed method is better than both method [5] and method [16] in terms of sensitivity, that is, the number of true-positive vessel identifications is high and has high agreement with the observers on pixels which are part of the vasculature, and the higher specificity
rates in comparison with the two methods suggests that our proposed method has a lower false-positive rate, i.e., it is labelling less pixels as vessel when the observers have not marked them as such.

Table 3 compares the average SE and SP on STARE database between our method and a few existing methods. We observed from the table that our method performs better. Table 4 shows the performances of our method on both DRIVE and STARE datasets with different methods, which have been described the main ideas in the introduction chapter for segmenting the retinal vasculature. The performances shown here are reported on www.isi.uu.nl/Research/ Databases/DRIVE/ or papers of the above authors. On STARE images, our proposed method is best in terms of ACC. Results with DRIVE images indicate that our method performs very well, but its ACC fall behind that of Lupascu et al. [14]. We emphasize here that our method using the binary level set method is equivalent to producing an adaptive thresholding surface based on the boundary information of MF enhanced image. So one advantage of our method is that smaller and weaker vessels can also be segmented well as long as their boundaries are relatively clear. No statistical information is involved as well. Due to the simplicity of our MF and the operator-splitting implementation technique in solving the PDE model, the computational cost of our method may be relatively low. Meanwhile, considering the fact that Lupascu et al. [14] build a 41-D feature vector for each pixel and Wang et al. [19] adopt a 2-D Hermite function intensity model as a modified Gaussian function, a kernel, which is more suitable with the cross-sections of real blood vessels, may possibly enhance the performance of our method further. We shall pursue this in future.

6. Conclusion

We have combined a few techniques for the problem of the retinal vessel segmentation. The initial blood vessel information is first enhanced by a matched filter function. Then the blood vessel segmentation may be done better by using the well-known binary level set method and the Mumford-Shah functional. To implement the method more efficiently, we apply a finite element and an operator-splitting technique to solve the Euler-Lagrange equation of the functional. In order to demonstrate the effectiveness and robustness of our method, we evaluate our method on one publicly available standard database of coloured images (with manual segmentations available too) and compare it with a few recent segmentation methods. The segmentation results show good performance of our method.

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Figure 1. Segmentation results produced by our method with different $n$ in Algorithm 1: (a) Im0162 image from the STARE data set; (b) the image enhanced by Equation (1); (c) Segmentation image $u$ at $n = 0$; (d) Segmentation image $u$ at $n = 50$; (e) Segmentation image $u$ at $n = 100$; (f) Segmentation image $u$ at $n = 150$; (g) Segmentation image $u$ at $n = 200$; (h) Segmentation image $u$ at $n = 250$; (i) Segmentation image $u$ at $n = 300$; (j) Segmentation image $u$ at $n = 350$; (k) Segmentation image $u$ at $n = 400$; (m) Segmentation image $u$ at $n = 450$. 
Figure 2. (a) Segmentation for image Im0162 from the STARE database using our method; (b) Result using the Hoover et al. ’s method[5]; (c) Manual segmentation from the first observer; (d) Manual segmentation from the second observer.

Figure 3. (a) Segmentation for image Im0162 from the STARE database using our method with parameter $\beta = 0.5$; (b) Segmentation for image Im0162 from the STARE database using our method with parameter $\beta = 50$. 
Figure 4. (a) Image 01 from the DRIVE data set; (b) Segmentation for image 01 using our method; (c) Manual segmentation from the first observer; (d) Manual segmentation from the second observer.

References

Table 1. Comparative Results of SE and SP on STARE Database Between Hoover [5] and Our method

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Table 2. Comparative Results of SE and SP on DRIVE Database between Staal [16] and Our method

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<tr>
<th>Image</th>
<th>Staal [16]</th>
<th>Our method</th>
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<tbody>
<tr>
<td></td>
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Table 3. Performance of Vessel Segmentation Methods

<table>
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<tr>
<th>Method</th>
<th>Data set</th>
<th>Avg. SE</th>
<th>Avg. SP</th>
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<tbody>
<tr>
<td>Our method</td>
<td>STARE</td>
<td>0.753</td>
<td>0.981</td>
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<tr>
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<td>STARE</td>
<td>0.734</td>
<td>0.976</td>
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<td>Wang et al. [19]</td>
<td>STARE</td>
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<td>0.980</td>
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### Table 4. Comparative Performance Summary

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<th>ACC</th>
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<tr>
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