Helicity Fluxes and Gauge Issues in Turbulent Dynamos

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Introduction

Quenching

Large scale magnetic fields in dynamos, like the Sun or Galaxy, are created on dynamical time scales.
In periodic boxes with helicity conservation dynamos grow only on resistive time scales (Brandenburg 2001).

Saturation magnetic energy varies as $R_m^{-1}$ which can be catastrophic for real
astrophysical objects (Sun: $R_m = 10^9$, Galaxy: $R_m = 10^{14}$). This catastrophic quenching
needs to be alleviated by helicity fluxes,
which will also give us fast growing dynamos.

As $R_m$ increases, larger fractions of magnetic
energy are stored in smaller scales.

\textbf{FIG. 1. Mean magnetic energy to the total magnetic energy versus magnetic
Reynolds number (Brandenburg & D"{o}rler 2001)}

Approach

Quenching

- 1D mean field simulation
- Linear forcing of kinematic helicity
- Allow for helicity fluxes out of the domain.
- Impose helicity fluxes through the equator.

The equations:

\[
\frac{\partial B}{\partial t} = -2\alpha B - \frac{2}{\alpha} \left( \frac{E \cdot B}{B^2} + \frac{\partial u}{\partial z} - \frac{\partial E}{\partial z} \right),
\]

\[
\frac{\partial B}{\partial t} = -\frac{\partial}{\partial z} (U \cdot B + \alpha B) + \frac{\partial E}{\partial z} B^2,
\]

\[
\frac{\partial B}{\partial t} = -\frac{\partial}{\partial z} (U \cdot B - \alpha B) + \frac{\partial E}{\partial z} B^2 + \frac{\partial}{\partial z} B^2.
\]

\textbf{FIG. 2. Saturation magnetic energy versus magnetic Reynolds number $R_m$ with
advective helicity flux (solid line). Compare the case without the flux (dashed line). The
catastrophic quenching gets alleviated by helicity fluxes.}

Approach

Gauging the Magnetic Helicity

Make direct numerical simulations (DNS) of compressible MHD for an isothermal gas
with constant sound speed.

Consider the gauges:
- Resistive gauge
- Lorentz gauge
- Weyl gauge

Small-scale and large-scale helicity changes:

\[
\frac{\partial H_m^{\text{iso}}}{\partial t} = -2 \mathbf{E} \cdot \mathbf{B} - 2 \mathbf{v}_g \cdot \mathbf{B} - \mathbf{\nabla} \cdot \mathbf{E}^{\text{iso}},
\]

\[
\frac{\partial H_{2,1}}{\partial t} = -2 \mathbf{E} \cdot \mathbf{B} - 2 \mathbf{v}_g \cdot \mathbf{B} - \mathbf{\nabla} \cdot \mathbf{E}_{2,1},
\]

Helicity fluxes:

\[
\mathbf{J}_m = \mathbf{E} \times \mathbf{A} + \mathbf{\Psi} \mathbf{B},
\]

\[
\frac{\partial}{\partial t} \mathbf{E} = \mathbf{\nabla} \times \mathbf{A},
\]

and $\mathbf{\Psi} = -\mathbf{\nabla} \times \mathbf{\Psi}$.

Fickian diffusion:

\[
\mathbf{E}^{\text{iso}} = -\kappa \mathbf{\nabla} H_m^{\text{iso}}.
\]

\textbf{FIG. 3. Saturation magnetic energy versus magnetic Reynolds number $R_m$ with
diffusive helicity fluxes through the equator (solid line). Compare the case without the flux (dashed line). Also here the quenching gets alleviated.}

Approach

Quenching

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\textbf{FIG. 4. Left Space-time diagrams for the mean values of $B_1$ and $B_2$. Right (upper): $z$-profile of the small and large scale helicities and their fluxes in a
simulation allowing for advective fluxes out of the domain. Right (lower): $z$-profile of the small and large scale helicities and their fluxes in a
simulation including Fickian diffusion through the equator.}

Results

Gauging the Magnetic Helicity

Equator ward migration

Fickian diffusion can model small scale helicity fluxes through the equator.

\textbf{FIG. 5. Bz component of the magnetic field at the periphery of the domain at different times. Note the equator ward migration of the large scale field.}

\textbf{FIG. 6. 2D dependence of the contributions to the helicity fluxes (upper two panels) and diffusive helicity flux (lower panel).}

Conclusions

- Magnetic helicity fluxes out of the domain can alleviate catastrophic quenching for high
magnetic Reynolds numbers.
- Magnetic helicity fluxes through the equator can also alleviate the quenching.
- Magnetic helicity fluxes in DNS are independent of the gauge.
- Magnetic helicity fluxes follow a Fickian diffusion law: $\kappa = 0.3\eta$.

References