Magnetic field topology and electric current formation in plasma.

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Magnetic Fields in Nature

Galaxies: 10e-6 G

Earth: 0.1-1G

Sun: 2-2,000G
\[ \beta = 0.01 \]
\[ R_m = 10^6 - 10^{12} \]

Pfrommer (2010)
Confining Plasma

ITER

Team TONUS (2014)

Large Helical Device, Toki (Japan)
Field's Environment in the Corona

Magnetically dominated:

magnetic pressure $\gg$ thermal pressure

$$\frac{B^2}{(2\mu_0)} \gg nk_B T$$

$$\beta = 2\mu_0 \frac{nk_B T}{B^2} \ll 1$$

Solar corona: $\beta \approx 0.01$

Frozen-in magnetic flux:

magnetic resistivity small: $t_{\text{dissipation}} \gg t_{\text{dynamical}}$

Magnetic field is \textit{frozen-in} to the fluid.

Batchelor (1950)
Topologies of Magnetic Fields

Hopf link

Borromean rings

twisted field

magnetic braid

trefoil knot

IUCAA knot
Magnetic Field Topology

Measure for the topology:

\[ H_M = \int_V A \cdot B \ dV = 2n\phi_1\phi_2 \]
\[ \nabla \times A = B \quad \phi_i = \int_{S_i} B \cdot dS \]

\( n = \text{number of mutual linking} \)

Conservation of magnetic helicity:

\[ \lim_{\eta \to 0} \frac{\partial}{\partial t} \langle A \cdot B \rangle = 0 \quad \eta = \text{magnetic resistivity} \]

Realizability condition:

\[ E_m(k) \geq k |H(k)|/2\mu_0 \]

Arnold (1974)

Magnetic energy is bound from below by magnetic helicity.

Moffatt (1969)
Interlocked Flux Rings
actual linking vs. magnetic helicity

$H_M \neq 0$

$H_M = 0$

- initial condition: flux tubes
- isothermal compressible gas
- viscous medium
- periodic boundaries

(Del Sordo et al. 2010)

$$\frac{\partial A}{\partial t} = U \times B + \eta \nabla^2 A$$
$$\frac{D \ln \rho}{Dt} = -\nabla \cdot U$$
$$\frac{D U}{Dt} = -c_S^2 \nabla \ln \rho + J \times B/\rho + F_{\text{visc}}$$
Interlocked Flux Rings

Magnetic helicity rather than actual linking determines the field decay.
Stability Criteria

Ideal MHD: \( \eta = 0 \)

Induction equation: \( \frac{\partial B}{\partial t} = \nabla \times (U \times B) \)

Woltjer (1958): \[ \frac{\partial}{\partial t} \int_V A \cdot B \, dV = 0 \]

Taylor (1974): \[ \frac{\partial}{\partial t} \int_{\tilde{V}} A \cdot B \, dV = 0 \]

Constraint

Equilibrium

\( \nabla \times B = \alpha B \)

\( \nabla \times B = \alpha(a, b)B \)

constant along field line

\( V \) total volume

\( \tilde{V} \) volume along magnetic field line
Taylor Relaxation

Field line magnetic helicity conservation

\[ \nabla \times \mathbf{B} = \lambda(a, b) \mathbf{B} \]

Taylor (1974)

Does the system always reach this state?

Yeates (2010)

Not necessarily. Additional topological degree must be conserved.
Force-Free Magnetic Fields

Solar corona: low plasma beta and magnetic resistivity

- Force-free magnetic fields
- Minimum energy state

\[(\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \iff \nabla \times \mathbf{B} = \alpha \mathbf{B}\]

**Parker:** Equilibrium with the same topology exists only if the twist varies uniformly along the field lines. Strongly braided fields → topological dissipation.

(Parker 1972)

Braided fields from foot point motion complex enough. (Parker 1983)

Solutions possible with filamentary current structures (sheets).

(Mikic 1989, Low 2010)
Methods

Ideal (non-resistive) evolution
Frozen in magnetic field

(use Lagrangian method)

Preserves topology and divergence-freeness.

Magneto-frictional term:
\[ \mathbf{u} = \mathbf{J} \times \mathbf{B} \]
\[ \mathbf{J} = \nabla \times \mathbf{B} \]
\[ \frac{dE_M}{dt} < 0 \]
(Craig and Sneyd 1986)

Fluid with pressure:
\[ \mathbf{u} = \mathbf{J} \times \mathbf{B} - \beta \nabla \rho \]

Fluid with inertia:
\[ \frac{d\mathbf{u}}{dt} = \left( \mathbf{J} \times \mathbf{B} - \nu \mathbf{u} - \beta \nabla \rho \right) / \rho \]

For \( \mathbf{J} = \nabla \times \mathbf{B} \) use mimetic numerical operators.
(Hyman, Shashkov 1997)

Own GPU code GLEMuR: (https://github.com/SimonCan/glemur)
(Candelaresi et al. 2014)
Distorted Magnetic Fields

resolved current concentrations

shear leads to strong currents

(Candelaresi et al. 2015)
Magnetic Nulls

Singular current sheets observed at magnetic nulls ($B = 0$)

(Syrovatskii 1971; Pontin & Craig 2005; Fuentes-Fernández & Parnell 2012, 2013; Craig & Pontin 2014)

\[ u = J \times B \]

\[ u = J \times B - \beta \nabla \rho \]

- singular current sheets at magnetic nulls
- Pressure cannot balance singularity.
Questions: How do disturbances travel into the domain? Reconnection at null point? Propagation in presence of nulls?

(Richard 2015)
E3 Experiments

full resistive MHD simulations with the PencilCode
initially homogeneous field, E3 type of boundary driving

Braid propagates into domain.
E3 Experiments

field line mapping

(Yeates et al. 2010)

Sun: field line connectivity with foot point motions
Magnetic Skeleton
Conclusions

- Topology preserving relaxation of magnetic fields.
- Current concentrations not singular.
- Current increases strongly with field complexity.
- Singular currents at magnetic nulls.
- Braiding through photospheric foot point motion.
- Null point disruption through boundary motions.
Simply Twisted Fields

Magnetic streamlines:

(Candelaresi et al. 2014)