Magnetic helicity transport in the advective gauge family

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- Advecto-resistive gauge
- Gauge transformation ($\Lambda$ method)
- Instability and its nature
- Helicity transport

Submitted to Physics of Plasma
Advective gauge

**Induction equation:**
\[
\frac{\partial B}{\partial t} = \nabla \times (U \times B - \eta J)
\]

**Resistive gauge**
\[
\frac{\partial A^r}{\partial t} = U \times B + \eta \nabla^2 A^r
\]

**Advecto-resistive gauge**
\[
\frac{\partial A^{ar}}{\partial t} = U \times B - \eta J - \nabla (U \cdot A^{ar} - \eta \nabla \cdot A^{ar})
\]
Instability

MHD equations

\[
\frac{DA_{i}^{ar}}{Dt} = -U_{j,i}A_{j}^{ar} + \eta \nabla^{2} A_{i}^{ar} \\
\frac{D \ln \rho}{Dt} = -\nabla \cdot U \\
\frac{DU}{Dt} = -c_{s}^{2} \nabla \ln \rho + \frac{c_{L}}{\rho} J \times B + F_{visc} + f
\]

advective derivative: \( \frac{D}{Dt} = \frac{\partial}{\partial t} + U \cdot \nabla \)

But: Advecto-resistive gauge is numerically unstable.
\( \Lambda \) method

- Work in the resistive gauge
- Make a gauge transformation
- Evolve also the gauge field

\[
\frac{DA^r_{i}}{Dt} = U_{j,i} A^r_{j} + \eta \nabla^2 A^r_{i}
\]

resistive gauge
\[
\frac{\partial A^r}{\partial t} = U \times B + \eta \nabla^2 A^r
\]


gauge transformation
\[
\Lambda_{i}^{ar} = A^r_{i} + \nabla \Lambda_{i}^{r:ar}
\]

evolve \( \Lambda \)
\[
\frac{DA^r_{i}}{Dt} = -U \cdot A^r_{i} + \eta \nabla^2 \Lambda_{i}^{r:ar}
\]
Normalized magnetic helicity versus time. The direct method becomes unstable already in the kinematic regime while the \( \Lambda \) method is inherently stable.
Nature of the instability

\[ \frac{DA_{i}^{ar}}{Dt} = -U_{j,i}A_{j}^{ar} + \eta \nabla^2 A_{i}^{ar} \]

\[ \nabla \times (\nabla \Lambda) \]

irrotational contributions to B and J

Lorentz force increases

velocity increases

\[ \text{crash} \]
Kinematic regime

Different spatial fluctuations for $h^r$ and $h^{ar}$

In the advecto-resistive gauge helicity transport becomes important for high Re

\[
\frac{k_1 h^{ar}}{B_{\text{rms}}^2} = c \text{Re}_M^{-a} \left( 1 + b \text{Re}_M^{2a} \right)
\]

\[
\eta \kappa_1^2 t
\]

\[
\frac{k_1 h_{\text{rms}}^s}{B_{\text{rms}}^2}
\]

\[
\frac{k_1 h_{\text{rms}}^r}{B_{\text{rms}}^2}
\]

\[
\frac{k_1 h_{\text{rms}}^s}{B_{\text{rms}}^2}
\]

\[
\frac{k_1 h_{\text{rms}}^r}{B_{\text{rms}}^2}
\]
Comparison with passive scalar

In the kinematic regime $h$ behaves like a passive scalar.

$h^{\text{ar}}$ has strong high-k tail

efficient turbulent cascade in the advecto-resistive gauge
Revisiting earlier works
Conclusions and Outlook

- Advecto-resistive gauge is unstable.
- $\Lambda$ method can be used universally.
- The advecto-resistive gauge efficiently makes magnetic helicity cascade to higher wave numbers.
- In the ar gauge magnetic helicity behaves like a passive scalar in the kinematic regime and for high Rm.

- Understand the high Rm hook for $h^\text{ar}$ better.