Decay of helical and non-helical magnetic knots

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Magnetic Helicity

\[ H_M = \int_V \mathbf{A} \cdot \mathbf{B} \ dV = 2n\phi_1\phi_2 \]

\[ \phi_i = \int_{S_i} \mathbf{B} \cdot d\mathbf{S} \]

Realizability condition:

\[ E_m(k) \geq k|H(k)|/2\mu_0 \]

Magnetic energy is bound from below by magnetic helicity.

\[ \text{magnetic helicity conservation} \quad \Re \mu \rightarrow \infty \quad \frac{dH_M}{dt} = 0 \]

Trefoil knot

Twisted field
Helical and non-helical setups

$H_M \neq 0$

$H_M = 0$

$H_M = 0$

Trefoil knot

IUCAA knot

Borromean rings

Compare with (Del Sordo et al. 2010):

$n=0$

$n=2$
Simulations

- $256^3$ mesh point
- Isothermal compressible gas
- Viscous medium
- Periodic boundaries

\[
\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}
\]
\[
\frac{\text{D} \mathbf{U}}{\text{D}t} = -c_S^2 \nabla \ln \rho + \mathbf{J} \times \mathbf{B} / \rho + \mathbf{F}_{\text{visc}}
\]
\[
\frac{\text{D} \ln \rho}{\text{D}t} = -\nabla \cdot \mathbf{U}
\]
Helicity of n-foil knots

\[ H_M = 2n_{app} \phi^2 \]

\[ n_{app} = H_M / 2\phi^2 \]

\[ H_M = (n_f - 2)n_f \phi^2 / 2 \]
Linking number

Sign of the crossings for the 4-foil knot

\[ n_{\text{linking}} = \frac{(n_+ - n_-)}{2} \]

Number of crossings increases like \( n_f^2 \)

\[ H_M \propto n_{\text{linking}} \]

\[ H_M \propto n_f^2 \]
Magnetic energy decay

Slower decay for higher $n_f$. 
Realizability condition

\[ \frac{2M(k)}{|H(k)|} \]

Realizability condition more important for high \( n_f \).
Helicity vs. energy

Knot is more strongly packed with increasing $n_f$.

Magnetic energy is closer to its lower limit for high $n_f$.

\[ E_M \propto l_{\text{knot}} \propto n_f \]

\[ H_M \propto n_f^2 \]
Magnetic helicity is approximately conserved.

Self-linking is transformed into twisting after reconnection.
Parametrization: \[ \mathbf{x}(s) = \begin{pmatrix} (C + \sin 4s) \sin 3s \\ (C + \sin 4s) \cos 3s \\ D \cos(8s - \varphi) \end{pmatrix} \]

Asymmetry due to parametrization
Field ejection

$B^2$

$t = 0$

$B^2$

$t = 100$

$\varphi = 4/3 \pi$

$\varphi = (4/3 + 0.2) \pi$
Field ejection

$t = 39$

Magnetic field ejection $\rightarrow$ Isolated helical structures

$t = 78$
Decay comparison

\[
\frac{\langle B^2 \rangle}{\langle B_0^2 \rangle} \quad t/t_{res}
\]

- **IUCAA knot**
- **Borromean rings**
- **helical triple rings**
- **non-helical triple rings**

\[ t^{-1/2} \] \[ t^{-1} \] \[ t^{-3/2} \]
Reconnection characteristics

Conversion of linking into twisting

Ruzmaikin and Akhmetiev (1994)
Reconnection characteristics

3 rings → Twisted ring + interlocked rings → 2 twisted rings

t = 70

t = 78
Conclusions

- Stronger packing for high $n_f$ leads to different decay slopes.
- Higher order invariants?

- Non-forced ejection of magnetic field
- Isolated helical structures inhibit energy decay
- Reconsider realizability condition
References

Del Sordo et al. 2010

Fabio Del Sordo, Simon Candelaresi, and Axel Brandenburg.
Magnetic-field decay of three interlocked flux rings with zero linking number.

Ruzmaikin and Akhmetiev 1994

A. Ruzmaikin and P. Akhmetiev.
Topological invariants of magnetic fields, and the effect of reconnections.