Decay of helical and non-helical magnetic links and knots
Magnetic helicity

\[ H_M = \int_V \mathbf{A} \cdot \mathbf{B} \, dV = 2n\phi_1\phi_2 \]

\[ \phi_i = \int_{S_i} \mathbf{B} \cdot d\mathbf{S} \]

Realizability condition:
\[ E_m(k) \geq k|H(k)|/2\mu_0 \]

Magnetic energy is bound from below by magnetic helicity.

magnetic helicity conservation
\[ \text{Re}_M \rightarrow \infty \]
\[ \frac{dH_M}{dt} = 0 \]
Interlocked flux rings

$H_M \neq 0$

$H_M = 0$

- Isothermal compressible gas
- Viscous medium
- Periodic boundaries
Interlocked flux rings

$$\tau = 4$$

$$H_M = 0$$

$$H_M \neq 0$$
Interlocked flux rings

Magnetic helicity rather then actual linking determines the field decay.
N-foil knots

3-foil  4-foil  5-foil  6-foil  7-foil

* from Wikipedia, author: Jim.belk

**cinquefoil knot**
Magnetic helicity is approximately conserved.

Self-linking is transformed into twisting after reconnection.
N-foil knots

Slower decay for higher $n_f$. 

$\langle B^2 \rangle / \langle B_0^2 \rangle$ vs $t/t_{res}$
N-foil knots

\[ H_M = 2n_1 \phi_2 \]

\[ n_{app} = \frac{H_M}{2\phi^2} \]

\[ H_M = (n_f - 2)n_f \phi^2 / 2 \]
Realizability condition more important for high $n_f$. 

\[ \frac{2M(k)}{|H(k)|} \]

\[ k = 2 \]
IUCAA knot and Borromean rings

- Is magnetic helicity sufficient?
- Higher order invariants?

IUCAA = The Inter-University Centre for Astronomy and Astrophysics, Pune, India
Reconnection characteristics

$t = 70$

3 rings → Twisted ring + interlocked rings → 2 twisted rings

$t = 78$
Reconnection characteristics

Conversion of linking into twisting

Ruzmaikin and Akhmetiev (1994)
Magnetic energy decay

\[ \frac{\langle B^2 \rangle}{\langle B_0^2 \rangle} \]

\[ t \rightarrow t_{res} \]

- IUCAA knot
- Borromean rings
- Helical triple rings
- Non-helical triple rings

\[ t^{-1/2} \]
\[ t^{-1} \]
\[ t^{-3/2} \]
Conclusions

- Topology can constrain field decay.
- Stronger packing for high $n_f$ leads to different decay slopes.
- Higher order invariants?

- Non-forced ejection of magnetic field
- Isolated helical structures inhibit energy decay
- Reconsider realizability condition
References

Candelaresi and Brandenburg 2011

Simon Candelaresi, and Axel Brandenburg.
Decay of helical and non-helical magnetic knots.
Phys. Rev. E, accepted.

Del Sordo et al. 2010

Fabio Del Sordo, Simon Candelaresi, and Axel Brandenburg.
Magnetic-field decay of three interlocked flux rings with zero linking number.

Ruzmaikin and Akhmetiev 1994

A. Ruzmaikin and P. Akhmetiev.
Topological invariants of magnetic fields, and the effect of reconnections.
Simulations

- $256^3$ mesh point
- Isothermal compressible gas
- Viscous medium
- Periodic boundaries

\[
\frac{\partial A}{\partial t} = U \times B + \eta \nabla^2 A
\]

\[
\frac{D U}{Dt} = -c_S^2 \nabla \ln \rho + J \times B / \rho + F_{\text{visc}}
\]

\[
\frac{D \ln \rho}{Dt} = -\nabla \cdot U
\]
Magnetic energy decay

\[ \frac{\langle B^2 \rangle}{\langle B_0^2 \rangle} \]

\[ \tau \]

\[ t^{-1/2} \]

\[ t^{-3/2} \]
Linking number

Sign of the crossings for the 4-foil knot

Number of crossings increases like $n_f^2$

$n_{\text{linking}} = (n_+ - n_-)/2$

$H_M \propto n_{\text{linking}}$

$H_M \propto n_f^2$
Helicity vs. energy

\[ E_M \propto l_{\text{knot}} \propto n_f \]
\[ H_M \propto n_f^2 \]

Knot is more strongly packed with increasing \( n_f \).

Magnetic energy is closer to its lower limit for high \( n_f \).