Magnetic Helicity and its Topological Interpretation

Nishant Singh\textsuperscript{1}, Simon Candelaresi\textsuperscript{2}

2010-01-11, Stockholm

\textsuperscript{1}Raman Research Institute, Bangalore
\textsuperscript{2}Nordita, Stockholm
Overview

1. Motivation
2. Magnetic Helicity
3. Linking Number
4. Simulations
5. Crossing Number
Motivation

- Differential rotation and turbulent convection may give rise to flux rings (alpha and omega effect) [Moffatt, 1983].
- Reconnection of magnetic fields in the corona $\rightarrow$ heating of the corona.
Magnetic Helicity

magnetic helicity:

\[ H_B = \int A B dV \]  \hspace{1cm} (1)

realizability condition:

\[ M(k) \geq k|H(k)|/2\mu_0 \] \hspace{1cm} (2)

The magnetic energy at each scale is bound from below.

[Brandenburg and Subramanian, 2005]
Linking Number

\[ F_1 = \int_{C_1} A \, dl \]  
\[ = \int_{S_1} \nabla \times A \, dS \]  
\[ = \int_{S_1} B_2 \, dS \]  
\[ = \phi_2 \]  

\[ \alpha_{ij} = \frac{1}{2\pi} \int_{C_i} \int_{C_j} \frac{\mathbf{R}(dl_i \times dl_j)}{R^3} \]  

\( \phi_i \) is the magnetic flux through the tube.

Figure 1: interlocked flux rings
Connection to linking number [Priest and Terry, 2000, p.266]:
mutual magnetic helicity:

\[ H_{Bmij} = \int A_i B_j \]  
(7)

\[ = 2 \alpha_{ij} \phi_i \phi_j \]  
(8)

other derivations on [Arnold, 1974].
⇒ Change of mutual helicity changes topology of the flux distribution.
Simulation Results

\begin{align*}
\langle B^2 \rangle / \langle B_0^2 \rangle & \quad t^{1/3} \\
B_{\text{max}} / B_B & \quad t^{3/2}
\end{align*}

Nishant Singh\textsuperscript{1}, Simon Candelaresi\textsuperscript{2}
Crossing number

\[ \text{linking number} = \frac{n_+ - n_-}{2} \]

