Magnetic Helicity Fluxes and their Effects in Dynamo Theory

Licentiate Thesis
Simon Candelaresi
2011-02-11
I. **Magnetic-field decay of three interlocked flux rings with zero linking number.**
Del Sordo F., Candelaresi S. and Brandenburg A.
*Phys. Rev. E, 81:036401, Mar 2010*

II. **Decay of trefoil and other magnetic knots.**
Candelaresi S., Del Sordo F. and Brandenburg A.
*arXiv:1011.0417*

III. **Small-scale magnetic helicity losses from a mean-field dyamo**
Brandenburg A., Candelaresi S. and Chatterjee P.

IV. **Equatorial magnetic helicity flux in simulations with different gauges.**
Mitra D., Candelaresi S., Chatterjee P., Tavakol R. and Brandenburg A.
*Astronomical Notes, 331:130-135, January 2010*

V. **Magnetic helicity transport in the advective gauge family.**
Candelaresi S., Hubbard A., Brandenburg A. and Mitra D.
*Physics of Plasmas, 18:012903, January 2011*
Introduction

11 year cycle

dynamo working
magnetohydrodynamics MHD

induction equation: \[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \mathbf{J}) \]

mean and fluctuating fields: \( \mathbf{B} = \overline{\mathbf{B}} + \mathbf{b} \)

\[ \partial_t \overline{\mathbf{B}} = \eta \nabla^2 \overline{\mathbf{B}} + \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathbf{E}}) \]

mean electromotive force (emf): \( \overline{\mathbf{E}} = \mathbf{u} \times \mathbf{b} \)

coupling with the mean field: \( \overline{\mathbf{E}} = \alpha \overline{\mathbf{B}} - \eta_t \nabla \times \overline{\mathbf{B}} \)
Introduction

\[ \alpha = \alpha_K + \alpha_M \]
\[ \alpha_K = -\tau \omega \cdot u / 3 \quad \alpha_M = \tau j \cdot b / (3 \rho) \]

Magnetic helicity density: \( h_M = A \cdot B \)

Helically driven dynamo:
\[ h_{K,f} = \omega \cdot u \]

Current helicity:
\[ h_{C,f} = j \cdot b \]

Production of magnetic helicity:
\[ h_{M,f} = a \cdot b \rightarrow -A \cdot B \]

\( a \cdot b \) works against dynamo:
\[ E_M \propto 1/Re_M \quad Re_M = \frac{UL}{\eta} \]

**Sun:** \( Re_M = 10^9 \)  
**Galaxies:** \( Re_M = 10^{29} \)
Realizability condition:

\[ E_m(k) \geq k |H(k)|/2\mu_0 \]

Magnetic energy is bound from below by magnetic helicity.

\[ \text{Re}_M \rightarrow \infty \]

\[ \frac{dH_M}{dt} = 0 \]

magnetic helicity conservation
Topological Interpretation

compressible isothermal fluid
periodic boundary conditions

\[ \text{Re}_M = \frac{UL}{\eta} = 10^3 \]

\[ \frac{dH_M}{dt} \approx 0 \]
$t = 4T_A$

$T_A = \sqrt{\mu_0 \rho_0 R_0^3 / \phi}$

$H = 0$

$H \neq 0$
$H_M = \int_V \mathbf{A} \cdot \mathbf{B} \, dV = 2n_{ij} \phi_i \phi_j$

$t = 5T_A$

$t = 0$

$\text{Re}_M = 100$
Magnetic energy decay

\[ \frac{\langle B^2 \rangle}{\langle B_0^2 \rangle} \]

- \[ t^{-1/2} \]
- \[ t^{-3/2} \]

\[ k = 4 \]

Magnetic helicity alone determines the field decay, not the actual linking.

Entangled fields are indistinguishable from non-entangled if the magnetic helicity is zero.
Topology Outlook

9-foil knot

\[ \int_V \mathbf{A} \cdot \mathbf{B} \, dV = 2 \alpha \phi \phi \]

valid?

IUCAA* logo

\[ H = 0 \]

Borromean rings

\[ H = 0 \]

*Inter-University Centre for Astronomy and Astrophysics, Pune, India
Dynamical alpha quenching

\[
\frac{\partial h_f}{\partial t} = -2E \cdot B - 2\eta \mu_0 j \cdot b - \nabla \cdot F_f
\]

\[
\frac{\partial \alpha_M}{\partial t} + \text{1d mean-field}
\]

vertical field (open boundaries)

increasing upward wind

symmetric field

0

perfect conductor (closed boundaries)

antisymmetric field

Fickian diffusion

\[ \kappa_\alpha \frac{\partial \alpha_M}{\partial z} \]

helical driving mechanism by \( \alpha_K \)
Dynamical alpha quenching

open boundary
symmetric
wind

VS.

closed boundary
antisymmetric

\( \kappa_{\alpha} \)

\[ \bar{F}_m \]

\[ \bar{F}_t \]

\[ \bar{B}_{\text{sat.}} / B_m \]

\[ C_U = 0.6 \]

\[ C_U = 0 \]

\[ 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \]

\[ 1.0 \quad 0.1 \quad 0.01 \quad 0.001 \quad 0.0001 \]

\[ \bar{B}_{\text{sat.}} / B_m \]

\[ \kappa_{\alpha} = 0 \]

\[ \kappa_{\alpha} = 0.05 \]

\[ 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \]
Gauge Issues

Gauge transformation: \( \mathbf{A} \rightarrow \mathbf{A} + \nabla \Lambda \)

\( H_m \rightarrow H_m + \int_S \Lambda \mathbf{B} \cdot dS \)

- resistive gauge
- pseudo-Lorenz gauge
- Weyl gauge

- helical forcing analog. MF
- periodic boundaries
- 128X128x256 box

Time averaged magnetic helicity fluxes do not depend on the gauge.

Its importance for dynamos is saved.
induction equation:
\[
\frac{\partial B}{\partial t} = \nabla \times (U \times B - \eta J)
\]

resistive gauge
\[
\frac{\partial A^r}{\partial t} = U \times B + \eta \nabla^2 A^r
\]

advecto-resistive gauge
\[
\frac{\partial A^{ar}}{\partial t} = U \times B - \eta J - \nabla (U \cdot A^{ar} - \eta \nabla \cdot A^{ar})
\]

measure helicity transport

spatial distribution of the magnetic helicity
But: Simulations are unstable. ☢️

Advevtive gauge

resistive gauge
\[ \frac{\partial A^r}{\partial t} = U \times B + \eta \nabla^2 A^r \]

gauge transformation
\[ A^{ar} = A^r + \nabla \Lambda \]

evolve \( \Lambda \)
\[ \frac{D \Lambda}{Dt} = -U \cdot A^r + \eta \nabla^2 \Lambda \]

\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + U \cdot \nabla \]
In the kinematic regime, $h^{ar}$ behaves like a passive scalar.

$\frac{DC}{Dt} = -\kappa \nabla^2 C$

$h^{ar}$ has strong high-k tail. Efficient turbulent cascade in the advecto-resistive gauge.
Magnetic helicity fluxes can alleviate catastrophic alpha quenching.

Diffusive fluxes within the domain can also alleviate catastrophic quenching.

Time averaged magnetic helicity fluxes are independent of the gauge.

The advecto-resistive gauge efficiently makes magnetic helicity cascade to higher wave numbers.

In the advecto-resistive gauge magnetic helicity behaves like a passive scalar in the kinematic regime and for high $Re_M$. 

Conclusions

- Magnetic helicity is the dominant quantity.
- No need (yet) for higher topological invariants.