Aim
Analysis plays a central role in mathematics and its applications, being fundamental for the existence and development of core mathematical fields, ranging from finite and infinite-dimensional dynamical systems (ODEs and PDEs), to numerical analysis, to statistics and probability, or to mathematical modelling in biological and physical sciences. In this module the concepts are defined precisely and the results are proved rigorously. In order for results to have broad application and the proofs to be liberated from unnecessary details, in most cases the topics included in this module are placed in a general context. While the module illustrates the logical development of a very important area of mathematics, this is an essential asset in the development of a robust mathematical curriculum.

Organisation
MA32001 is a Semester 1 module. Pre-requisites are MA21001 and MA22002.

The Module Leader is:
Dr Dumitru Trucu,
Room: G7, Fulton Building,
Phone: 01382 384462,
Email: trucu@maths.dundee.ac.uk

Timetable
The course has a total of 33 classes that are organised in 25 lectures and 8 tutorials. These will take place at the following times and locations:

- **Tuesdays** from 09:00 to 10:00, in J1 (Fulton Building);
- **Wednesdays** from 10:00 to 11:00, in Room 2.08 (Scrymgeour Building);
- **Thursdays** from 13:00 to 14:00, in J1 (Fulton Building).

Learning outcomes
By the end of the course you should know and understand the precise definitions and key theorems regarding various concepts related to: metric spaces, convergence, continuity, open and closes sets, compactness, connectedness, uniform continuity, equicontinuity of family of functions. You will be exposed to rigorous proofs of important results concerning these concepts, which will ultimately enable you to perform robust theoretical explorations of the complex connections between these key mathematical notions.
Syllabus

- Supremum, Completeness Axiom [2 lectures]
- Definitions and properties of normed and metric spaces, convergence of sequences, continuity, closed sets (in terms of limit points) [5 lectures]
- Connected sets: definition in metric spaces; relation to the concept of continuity [1 lecture]
- Cauchy sequences, completeness and relation to closed sets, Banach’s contraction mapping theorem [3 lectures]
- Compact sets:
  - general definition with open sets of the notion of compactness, its sequences characterisations on metric spaces, and its connection with closed subsets [2 lectures]
  - “closed an bounded” characterisation of the compact sets in $\mathbb{R}^n$ (i.e., the Heine-Borel Theorem), connection with limit points, Weierstrass Theorem [1 lecture]
  - connection between continuity and compactness [1 lecture];
  - the concept of uniform continuity and its connection with compactness [1 lecture];
  - Urysohn’s Lemma [1 lecture];
  - Partition of Unity on $\mathbb{R}^n$ [1 lecture];
- Uniform convergence of sequences of functions [2 lectures]
- The concept of equicontinuity of a family of functions and Arzela-Ascoli Theorem [1 lecture];
- Series: ratio test, comparison test, Weierstrass M-test [1 lecture];
- Power series, Taylor series (mention of Taylor’s theorem) [3 lectures];

Assessment

During the semester, there will be four compulsory homework problem sets, which will be offered at the beginning of the 3rd, 5th, 7th, and 9th week of lectures, as well as one in-class test (midterm).

The students are required to scan the solutions to each of the four compulsory homeworks and submit them via the Blackboard electronic homework submission procedure. The solutions for each of these homeworks must be submitted within one week of their release, namely by the beginning of the 4th, 6th, 8th, and 10th week, respectively. These homework problem sets will be graded, and each of them will contribute with 4% towards the final module assessment.

The in-class test (midterm) will take place in one of the days during the weeks 8 or 9 of the semester, at a date to be commonly agreed upon. The grade achieved in the in-class
test (midterm) will also contribute with 4% towards the final module assessment.

Finally, the assessment procedure will be completed with a Degree Examination in December 2015.

Therefore, the overall module assessment has the following distribution:

<table>
<thead>
<tr>
<th>Assessment Type</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compulsory homework problem set 1</td>
<td>4%</td>
</tr>
<tr>
<td>Compulsory homework problem set 2</td>
<td>4%</td>
</tr>
<tr>
<td>Compulsory homework problem set 3</td>
<td>4%</td>
</tr>
<tr>
<td>Compulsory homework problem set 4</td>
<td>4%</td>
</tr>
<tr>
<td>In-class test (midterm)</td>
<td>4%</td>
</tr>
<tr>
<td>Degree Examination in December 2015</td>
<td>80%</td>
</tr>
</tbody>
</table>

The Head of Division may debar a student not performing at a satisfactory level in the continuous assessments from entering the Degree Examination. A combined mark of 40% is required to pass the module.

**Your Commitment**

You should attend all lectures and tutorials except on medical grounds or with the special permission of the lecturer concerned.

**Study Support**

If you are having difficulty with the course work you are encouraged to seek help at an early stage at the tutorials.

**Feedback**

At the end of the module you will be asked to complete a confidential questionnaire regarding the content and presentation of the module. This is an important element in the University’s Academic Standards procedures.

**Primary references**

The course is provided with a complete MA32001 Lecture Notes that will be made available in a PDF on the Blackboard System. The MA32001 Lecture Notes PDF will be updated after each class with the material covered in that
particular lecture, and so, by the end of the semester, this will gradually build up in a unitary manuscript containing rigorous proofs and completely discussed examples covering the entire module.
All the theoretical material necessary to prepare for the final examination and the homeworks will be completely covered and fully explained in the MA32001 Lecture Notes.

For further reading on the topics included in this module, please consult the books from the module **Reading List**, namely:


**Supplementary references**
For advanced reading, I highly recommend the following books:

- D. W. Stroock, Essentials of Integration Theory for Analysis, (Springer, 2011);
- E. M. Stein and G. Weiss, Introduction to Fourier Analysis on Euclidean Spaces, (Princeton University Press, 1971);
- P. D. Lax, Functional Analysis (Wiley-Interscience, 2002);
- K. Yosida, Functional Analysis, Sixth Edition (Springer, 1980);