Organisation

The module runs for 11 teaching weeks in the first semester, and is worth 10 SCQF credits (equal to 5 ECTS points). The contact information for the module leader and lecturer, who is responsible for the entire organisation and teaching of the module, is below:

Prof. Ping Lin,

**Location:** Mathematics Division,
room J16, Fulton Building

**Tel:** 01382 384473

**email:** plin [at] maths [dot] dundee [dot] ac [dot] uk

**WWW:** http://www.maths.dundee.ac.uk/plin

Timetable

There will be 2 hours of class time each week. We will apply a hands-on, interactive approach, where lectures are integrated with guided computer lab time.

Each week will begin with lecture and time for student questions. The remainder of class session will be dedicated to guided investigations to solve physical problems using MATLAB, COMSOL, and related computational tools. At the end of class, computational problems will be assigned to assess your mastery of the lecture and tutorial content. The problems, complete with well-commented source, are due at the beginning of the next session. Marks will be deducted from late solutions.

Assessment

The overall assessment will primarily be based on your marks for the computational problem sets (100%).

Your Commitment

You are expected to attend all lectures / tutorials except on medical grounds or with the special permission of the lecturer. If you are unable to attend the degree examination or complete elements of the coursework (e.g. turn in homework solutions) on time on medical grounds, you should inform the lecturer and **submit a medical certificate to your Faculty Office.**
Study Support

If you are having difficulty with the coursework you should make an appointment to see the lecturer. Flexible office hours are available by schedule.

Syllabus

Prerequisites include a 2.2 honours degree in a relevant mathematical discipline, or equivalent.

In this course, we will learn to apply built-in “black box” solvers in MATLAB and COMSOL to mathematical modelling problems. To better understand the inner workings of these tools, students will also learn to “hand-code” basic ODE and PDE solution methods. Students will apply these skills to develop computational modelling techniques that extend beyond the standard built-in MATLAB and COMSOL solvers.

1. **MATLAB fundamentals:** Students will learn basic operations in MATLAB, and implement various finite difference schemes to solve ODEs (primarily initial value problems) originating in celestial mechanics, population dynamics, and cell biomechanics.

2. **MATLAB ODE solvers for initial value problems:** Students will learn to use standard built-in solvers with MATLAB, particularly ode45 and ode23s, and possibly dde23. We will apply these solvers to initial value problems (and possibly delay differential equations) stemming from celestial mechanics, cell biomechanics, and population dynamics.

3. **MATLAB random variables, stochastic processes, and SDEs:** After a brief introduction to stochastic differential equations (SDEs), students will learn MATLAB solution techniques, with applications to Brownian motion and related physical processes. We will also learn to simulate discrete and continuous stochastic processes, and generate samples from random variables with arbitrary distributions.

4. **MATLAB ODE solvers for boundary value problems:** Students will implement a standard “shooting” method to solve a BVP from heat transfer. We will learn to use the standard built-in solvers, particularly bvp4c. We explore alternate solution techniques, such as by formulating the discretised equation as a linear algebraic system, and as the steady state solution to a PDE; these approaches help drive us towards PDE solution methods. The class will apply these solvers to boundary value problems stemming from heat transfer and fluid mechanics.

5. **MATLAB for PDEs:** Students will implement explicit finite difference methods in MATLAB, with a focus on reaction-diffusion problems. The overall goal will be to solve coupled reaction-diffusion problems combining substrate transport (with heterogeneous coefficients) and cell growth.
6. **Weak formulations for partial differential equations; introduction to FEMs:**
   We repose PDEs using a weak formulation, using the context of function spaces. Using this framework, we develop an understanding of finite element methods (FEMs).

7. **FEMs and COMSOL fundamentals:** Students will learn to solve reaction-diffusion equations using the built-in FEMs in COMSOL.

8. **Optional topics:** As time permits, and by student interest, we will explore further computational modelling topics:
   
   (a) Delay differential equations and their roles in biochemistry and cell signalling
   (b) Promoter-inhibitor systems
   (c) Cell differentiation and stem cell dynamics
   (d) Variable geometry in COMSOL and MATLAB
   (e) Ghost fluid methods for discretising across irregular domains (or for discontinuous coefficients)
   (f) Geometric operations in image processing
   (g) Integro-differential equations (non-local cell-cell adhesion in continuum models)
   (h) Group mini-projects
   (i) Open source software alternatives.

**Feedback**

The module leader appreciates your feedback throughout the duration of the semester, and will endeavour to incorporate it into future lesson planning.

At the end of the module you will be asked to complete a confidential questionnaire regarding the content and presentation of the module. This is an important element in the University’s Academic Standards procedures.

You may bring matters of concern about the course to the attention of the Mathematics Division Staff/Student Committee, which meets once each semester. A volunteer will act as the MSc representative to sit on the Staff-Student Committee; their name will be posted on BlackBoard.

**Recommended Books and References**

There are a number of books on numerics and scientific computing which cover the material in the course and beyond. The following books and references may be particularly helpful:

2. Mathsoft MATLAB website:


4. Des Higham’s website (good for stochastic ODEs):
   http://personal.strath.ac.uk/d.j.higham/

5. A good paper on numerical stochastic ODEs:
   http://epubs.siam.org/sam-bin/dbq/article/37830


7. B. Øksendal, Stochastic Differential Equations: An Introduction with Applications,

8. HIPR2: a nice image processing algorithm website from the days of yore:
   http://www.dai.ed.ac.uk/HIPR2/