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Kinematic reconnection at a magnetic null point: fan-aligned current

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Magnetic reconnection at a three-dimensional null point is a natural extension of the familiar two-dimensional X-point reconnection. A model is set up here for reconnection at a null point with current directed parallel to the fan plane, by solving the kinematic, steady, resistive magnetohydrodynamic equations in its vicinity. The magnetic field is assumed to be steady, and a localised diffusion region surrounding the null point is also assumed, outside which the plasma is ideal. Particular attention is focussed on the way that the magnetic flux changes its connections as a result of the reconnection. The resultant plasma flow is found to cross the spine and fan of the null, and thus transfer magnetic flux between topologically distinct regions. Solutions are also found in which the flow crosses either the spine or fan only.

Keywords: Magnetic reconnection; Magnetohydrodynamics; Magnetic flux; Magnetic null points

1. Introduction

Magnetic reconnection is a fundamental process in many areas of plasma physics, whereby the magnetic field (**B**) becomes restructured. When null points are present the global topology of the field changes. Our ideas on how this restructuring occurs come mostly from the well-studied case of reconnection in two dimensions. In two dimensions, reconnection occurs at hyperbolic null points of the magnetic field (see, e.g., Priest and Forbes, 2000, for a review), commonly known as X-points (see figure 1). A plasma flow transports magnetic field lines towards the X-point, where field lines carried in from both sides of the null point break, at the moment that they lie along the separatrics of the field. The flow then transports the reconnected magnetic field lines, whose footpoints are now pairwise oppositely connected, away from the X-point.

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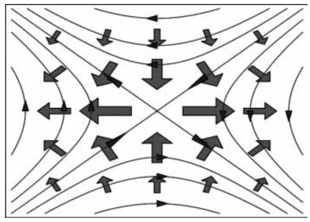

Figure 1. Two-dimensional reconnection at an X-point. The thin lines are magnetic field lines and the bold arrows indicate the direction of the plasma flow.

Most astrophysical plasmas are effectively ideal, in that any non-ideal terms are negligible except in very small regions. In an ideal plasma, magnetic field lines maintain their identity for all time, and are said to be 'frozen-into' the plasma. In other words, all plasma elements connected by a given field line at a given time will remain on that field line for all subsequent time. However, magnetic reconnection is a fundamentally non-ideal process. If the non-ideal behaviour is the result, for example, of a non-zero resistivity, η, then, assuming no other non-ideal effects are important, the process satisfies Ohm's law in the form

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}. \tag{1}$$

In order to investigate the evolution of magnetic flux in two dimensions, a flux transporting velocity $\mathbf{w}$ (Hornig and Schindler, 1996; Hornig and Priest, 2003) may be defined, which satisfies

$$\mathbf{E} + \mathbf{w} \times \mathbf{B} = \mathbf{0}. \tag{2}$$

This is always possible in two dimensions since the electric field, $\mathbf{E}$, and magnetic field, $\mathbf{B}$, are perpendicular. The component of $\mathbf{w}$ perpendicular to $\mathbf{B}$ is given by

$$\mathbf{w}_{\perp} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}. \tag{3}$$

If the non-ideal term on the right-hand side of (1) is localised within a finite region, D, about the null point, then outside D, $\mathbf{w}$ coincides with the plasma velocity perpendicular to $\mathbf{B}$, $\mathbf{v}_{\perp}$. Note that for reconnection to take place, the flux transporting velocity $\mathbf{w}$ must become singular at the null point, which is a signature of the breaking of the field lines (Hornig, 2001), and the subsequent discontinuity in the mapping of their endpoints.

The situation in three dimensions is much more complicated. In general for reconnection in three dimensions $\mathbf{E} \cdot \mathbf{B}$ (or $\mathbf{J} \cdot \mathbf{B}$) is non-zero, and hence in general no unique flux-conserving velocity (field line velocity) exists (Hornig and Schindler, 1996; Priest *et al.*, 2003). Nonetheless, it is still possible to study the evolution of magnetic flux and field lines if the non-ideal process is localised within a finite region, D. A finite non-ideal region is, in any case, the generic situation for astrophysical plasmas, since these plasmas have extremely high magnetic Reynolds numbers, and dissipation is enhanced only in very small regions where e.g. the presence of a thin current sheet may result in the formation of micro-instabilities.

Even though no unique field line velocity exists, the motion of individual magnetic field lines can still be followed, so long as no closed field lines exist within D, since we know that in the ideal region on either side of D they must remain attached to the same plasma elements for all time. So, by following the motion of field lines, anchored in the ideal region, we can define two flux velocities, one with which the field lines passing *into* D move, say $\mathbf{w}_{\text{in}}$, and another velocity $\mathbf{w}_{\text{out}}$ at which the field lines passing *out of* D move. In contrast to the situation in two-dimensions, these two velocities are not identical to each other inside D, nor are they identical to $\mathbf{v}_{\perp}$ on their continuations through D. This is a manifestation of the non-existence of a unique flux-conserving velocity, $\mathbf{w}$, as stated above. The result is that if a single field line is traced from both ends as it is transported towards D, then as soon as it is transported into the diffusion region it seems to split, and inside D the two field lines continually change their connections.

Reconnection can occur in three dimensions either at a null point or in the absence of a null point (Schindler *et al.*, 1988). The nature of magnetic reconnection in the absence of a three-dimensional null point has been discussed by Hesse (1991) and Horing and Priest (2003). It was found that the plasma velocity $\mathbf{v}$ outside D has a rotational structure. In a previous paper (Pontin *et al.*, 2004, hereafter referred to as Paper I) we have discussed the nature of reconnection at a three-dimensional null point with current parallel to the spine axis. In this case a rotational type of flux reconnection was also found. In both solutions, regions of the flow are found where a classical type of reconnection occurs, such that field line footpoints which are initially joined to each other become newly connected to separate footpoints in completely different regions of the flow, and move apart for all times. Moreover, neither reconnection process allows for a simple one-to-one reconnection of field lines, so that specific pairs of footpoints can never become pairwise oppositely connected after the reconnection process is complete, as is the case in two dimensions. In this paper, we study the case of a three-dimensional null point whose current is aligned with its fan. Although many of the general properties described in Paper I are again observed here, the structure of the plasma flow, and so the nature of the field line motion, is very different.

In section 2 we discuss three-dimensional null points and ideal field line behaviour in their vicinity. We describe our model in section 3, and in section 4 the basic solution. The existence of perfectly reconnecting field lines is discussed in section 5, and in section 6 we extend our analysis to looking at situations where the current is non-constant. Section 7 contains our conclusions.

2. Structure of 3D Null Points and ideal behaviour

The nature of reconnection at a three-dimensional null point is of great interest since it is the three-dimensional analogue of a 2D X-point. Three-dimensional nulls are also of crucial importance in the topology and interaction of complex fields on the Sun. They are found in abundance in the solar corona (see, eg., Schrijver and Title, 2002; Longcope. *et al.*, 2003), where their associated separatrices and separators are thought to be likely candidates for sites of coronal heating (Longcope, 1996; Antiochos *et al.*, 2002; Priest *et al.*, 2002). There is also evidence that null point reconnection may act as a trigger for at least some solar flares (Fletcher *et al.*, 2001).

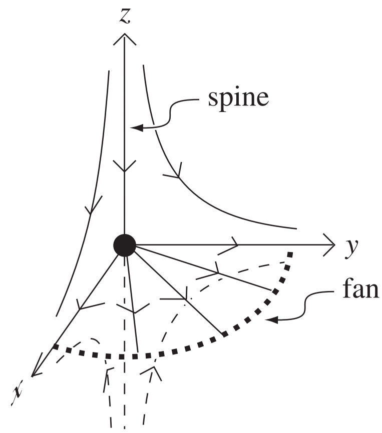

Figure 2. The local structure of a 3D null point.

The local structure of the magnetic field around a three-dimensional null point is shown in figure 2. The *skeleton* of the null point is made up of a pair of field lines directed into (or out of) the null from opposite directions, known as the *spine*, and a family of field lines which are directed out of (or into) the null lying in a surface, known as the *fan* plane (Priest and Titov, 1996). A general mathematical formalism is given by Parnell *et al*. (1996), who classify nulls depending, amongst other things, on the size of the current, **J**, and its direction with respect to the spine axis and fan plane. If the current is zero, then the null point is known as *potential*. A current component parallel to the spine results in a spiralling of the field lines in the fan, while the fan and spine are perpendicular. Reconnection at a null point with this current structure was discussed in Paper I. By contrast, a current component parallel to the fan plane results in the spine and fan folding up such that they are no longer perpendicular. It is a null with this current structure that we will consider in this paper. In general, a null point with current components in both directions will exhibit both types of behaviour.

The kinematics of steady reconnection at an isolated three-dimensional null point have been studied previously by Priest and Titov (1996). They start off by discussing the ideal behaviour in the vicinity of the simple potential magnetic null given by

$$\mathbf{B} = (x, y, -2z). \tag{4}$$

The reconnection is classified as one of two types near an isolated null point, termed *spine reconnection* and *fan reconnection*. Since the configuration considered is ideal, the plasma velocity is necessarily singular at the null to achieve reconnection. During spine reconnection a flow is imposed across the fan ($z = 0$) resulting in singularities in **E** and **v** at the spine ($x = y = 0$). In fan reconnection, the flow is imposed across the spine resulting in singularities in **E** and **v** in the fan plane. The effect of adding non-potential and diffusive terms is then considered in a preliminary manner.

We aim here to investigate the structure of reconnection when the null point is non-potential, with a current parallel to the fan plane, where a localised diffusion region is included in order to get a realistic model with no singularities in any physical quantities. In order to investigate the nature of the restructuring of the magnetic flux in the reconnection process, we follow the method of Horing and Priest (2003) and Pontin *et al*. (2004) in adopting a kinematic approximation, that is, we consider here only the effects of the induction equation and Maxwell's equations. We intend to go on to investigate at a later date, with the help of numerical models, whether the properties we find for these complex processes carry through when the full set of MHD equations is included. In a fully dynamical situation we would expect to start from an initial field, which then forms a current sheet (diffusion region) at which reconnection takes place. The way in which a current sheet may form by the collapse of a three-dimensional null point is discussed by Parnell *et al*. (1997) and Mellor *et al*. (2003) as well as by Klapper *et al*. (1996) and Bulanov and Sakai (1997).

3. The model

We seek a solution to the kinematic, steady, resistive MHD equations in the locality of a magnetic null point. That is, we solve

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}, \tag{5}$$

$$\nabla \times \mathbf{E} = \mathbf{0}, \tag{6}$$

$$\nabla \cdot \mathbf{B} = \mathbf{0}, \tag{7}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}. \tag{8}$$

Whereas in Paper I we examined the kinematic behaviour around a magnetic null point whose associated current was parallel with its spine axis, here we consider a null point with current directed parallel to the fan plane. We choose

$$\mathbf{B} = B_0(x, y - jz, -2z), \tag{9}$$

such that, without loss of generality, the current lies in the x-direction, and is given by $\mathbf{J} = (\nabla \times \mathbf{B})/\mu_0 = (B_0/\mu_0)(j, 0, 0)$, from (8). The fan plane of this magnetic null point is coincident with the plane $z = 0$, while the spine is not perpendicular to this, but rather lies along $x = 0$, $y = jz/3$ (see figure 3).

For the chosen magnetic field (9), analytical expressions for the equations of magnetic field lines can be found, by solving

$$\frac{\partial \mathbf{X}(s)}{\partial s} = \mathbf{B}(\mathbf{X}(s)), \tag{10}$$

where the parameter s runs along field lines, and is related to the length, $\mathrm{d}\lambda$, along a field line by

$$\mathrm{d}\lambda = |B| \mathrm{d}s. \tag{11}$$

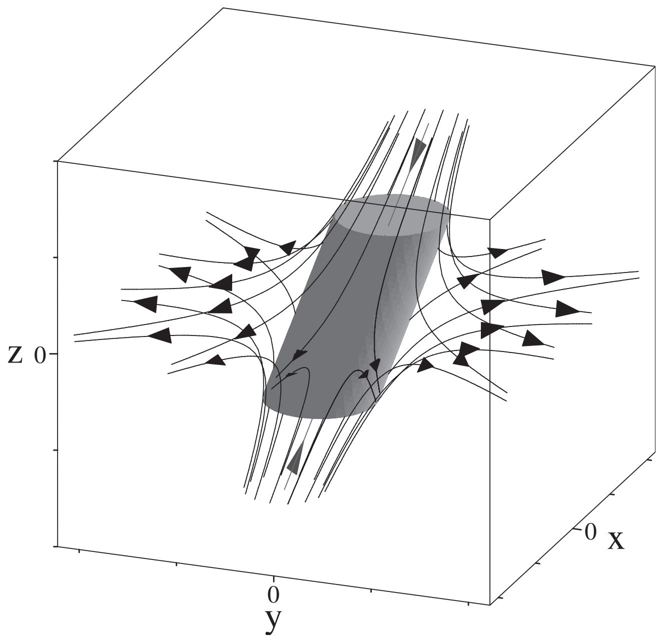

Figure 3. The basic structure of the 3D null point studied. The fan plane is in the $z = 0$ plane, while the spine line (grey) lies along $x = 0$, $y = jz/3$. The shaded cylinder shows the shape of the diffusion region.

Solving equation (10) we obtain

$$x = x_0 e^{B_0 s}, \tag{12a}$$

$$y = \tfrac{1}{3} j z_0 e^{-2B_0 s} + \left(y_0 - \tfrac{1}{3} j z_0\right) e^{B_0 s}, \tag{12b}$$

$$z = z_0 e^{-2B_0 s}, \tag{12c}$$

which describes the equations of the magnetic field lines in terms of some initial coordinates $\mathbf{X}_0 = (x_0, y_0, z_0)$.

We proceed to solve (5)–(8) as follows. From equation (6) we can write, in general, $\mathbf{E} = -\nabla\Phi$, where Φ is a scalar potential. Then the component of equation (5) parallel to $\mathbf{B}$ is $-(\nabla\Phi)_\parallel = \eta J_\parallel$, and so we can calculate Φ by integrating along magnetic field lines,

$$\Phi = -\int \eta \mathbf{J}\cdot\mathbf{B}\,\mathrm{d}s + \Phi_0, \tag{13}$$

where Φ_0 is a constant of integration. We now follow Paper I in choosing the particular profile of η such that this integration can be performed analytically. By first substituting equations (12) into the integrand, we can integrate equation (13) to obtain $\Phi(\mathbf{X}_0, s)$, after which we can use the inverse of equation (12) to find $\Phi(\mathbf{X})$. The electric field can now be found

$$\mathbf{E} = -\nabla\Phi, \tag{14}$$

as can the plasma velocity perpendicular to the magnetic field $\mathbf{v}_\perp$, by taking the vector product of equation (1) with $\mathbf{B}$ to obtain

$$\mathbf{v}_\perp = \frac{(\mathbf{E} - \eta\mathbf{J}) \times \mathbf{B}}{B^2}. \tag{15}$$

Now, in the simple model described above, the electric current, $\mathbf{J}$, is constant, and we would like to consider a non-ideal region which is localised in space, so we prescribe the resistivity to be localised, of the form

$$\eta = \eta_0 \begin{cases} \left[\left(\dfrac{R_1}{a}\right)^2 - 1\right]^2 \left[\left(\dfrac{z}{b}\right)^2 - 1\right]^2 & R_1 < a, \quad z^2 < b^2, \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $R_1^2 = x^2 + (y - jz/3)^2$ and η_0, a and b are constants. η_0 is the value of η at the null point, and the diffusion region is a tilted cylinder centred on the spine axis, extending to $z = \pm b$ and with radius a. This is shown, together with the structure of the magnetic field, in figure 3. The diffusion region is chosen to be of this shape to simplify the calculations described above.

In order to perform the integration in equation (13) we must choose a suitable surface on which to set $s = 0$ and so start the integration. The surface should be chosen such that it intersects every field line once and only once so that the mapping $(\mathbf{X}_0, s) \to (\mathbf{X})$, given by equation (12), is one-to-one. With this in mind, we set $s = 0$ on $z = \pm z_0$.

Performing the integration of equation (13) setting $s = 0$ on $z = z_0$ now gives an expression for $\Phi(\mathbf{X}_0, s)$ for $z > 0$, and setting $s = 0$ on $z = -z_0$ gives $\Phi(\mathbf{X}_0, s)$ for $z < 0$. In order for these two expressions to match in the fan plane, that is for Φ to be smooth and continuous, and thus physically acceptable, we must set the value of Φ at $z = \pm z_0 (\Phi_0)$ to be

$$\Phi_0 = \frac{32}{21} \eta_0 B_0 \, j \, x_0. \tag{17}$$

This is equivalent to ensuring that when field lines in the fan plane converge on the null point, Φ approaches the same value along them, so that it is single-valued at the null. $\Phi(\mathbf{X})$, $\mathbf{E}$ and $\mathbf{v}_\perp$ can now be obtained from (13)–(15) using (12) and (16), as described earlier. The mathematical expressions are too lengthy to show here, but can be calculated in a straightforward way using a symbolic computation package.

4. Nature and rate of reconnection

In order to study the effect of the reconnection on the magnetic flux, we examine the plasma velocity perpendicular to $\mathbf{B}$ ($\mathbf{v}_\perp$). This is the flux velocity in the ideal region, and is thus the component of the velocity that affects the flux transport. The nature of this flow in a plane of constant x is shown in figure 4. The flow in the x-direction is zero across the spine, and is negligible for the reconnection process. The plasma

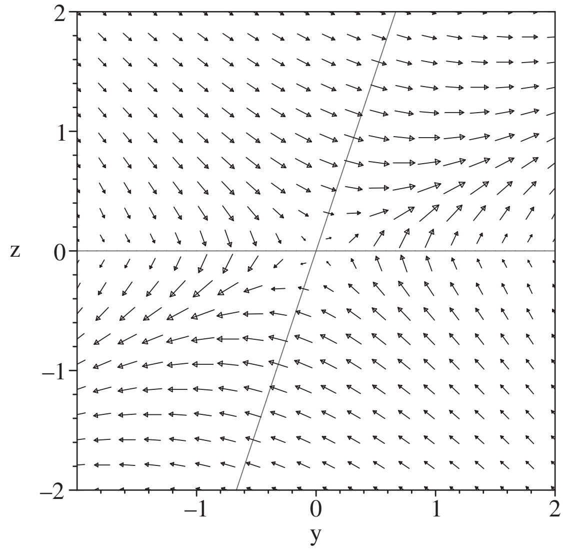

Figure 4. The structure of the plasma flow, along with the fan and spine (black lines) in a typical plane of constant x, for parameters $\eta_0 = B_0 = j = a = b = 1$.

flow is non-zero across both the spine and the fan, having a stagnation point structure in the *yz*-plane, centred on the null point. Note that this is very different from the situation described in Paper I, where **J** is parallel to the spine of the null point, and there is no flow across either the spine or the fan. The result of the plasma flows across the spine and fan is that the nature of the field line behaviour under the reconnection is qualitatively the same as described by Priest and Titov (1996) in the ideal analysis, with field lines advected across the spine having a behaviour like the *fan reconnection*, and those advected across the fan having a *spine reconnection* type behaviour. As shown in this section, these two types of behaviour are not mutually exclusive. In section 6, solutions are described which de-couple them.

Consider following field lines, with footpoints anchored in the ideal region, which pass across the top of the diffusion region D, such that they pass down through D. These field lines can be seen to flip around the spine in the fan plane, as illustrated in figure 5, with a behaviour analogous to the fan reconnection of Priest and Titov (1996). By contrast, following field lines anchored in the flow across the fan plane we see these field lines move towards the fan until they lie in the fan plane, at which point they flip down the spine, and then move away from the null on the opposite side of the fan (analogous to spine reconnection).

Although a continuous stream of field lines is reconnected through the spine, no finite amount of flux is reconnected at it, since it is a single line. A finite amount of flux *is*, however, transported across the fan plane in an arbitrary time. This can be very important, since if we consider our null point to be surrounded by a global field, for example in the solar corona, then the fan plane is a separatrix surface of this magnetic field, which separates distinct regions of magnetic topology. The topology of this field changes when flux is transported across a separatrix surface. As described previously, in the solar atmosphere separatrices are thought to be very important locations for coronal heating.

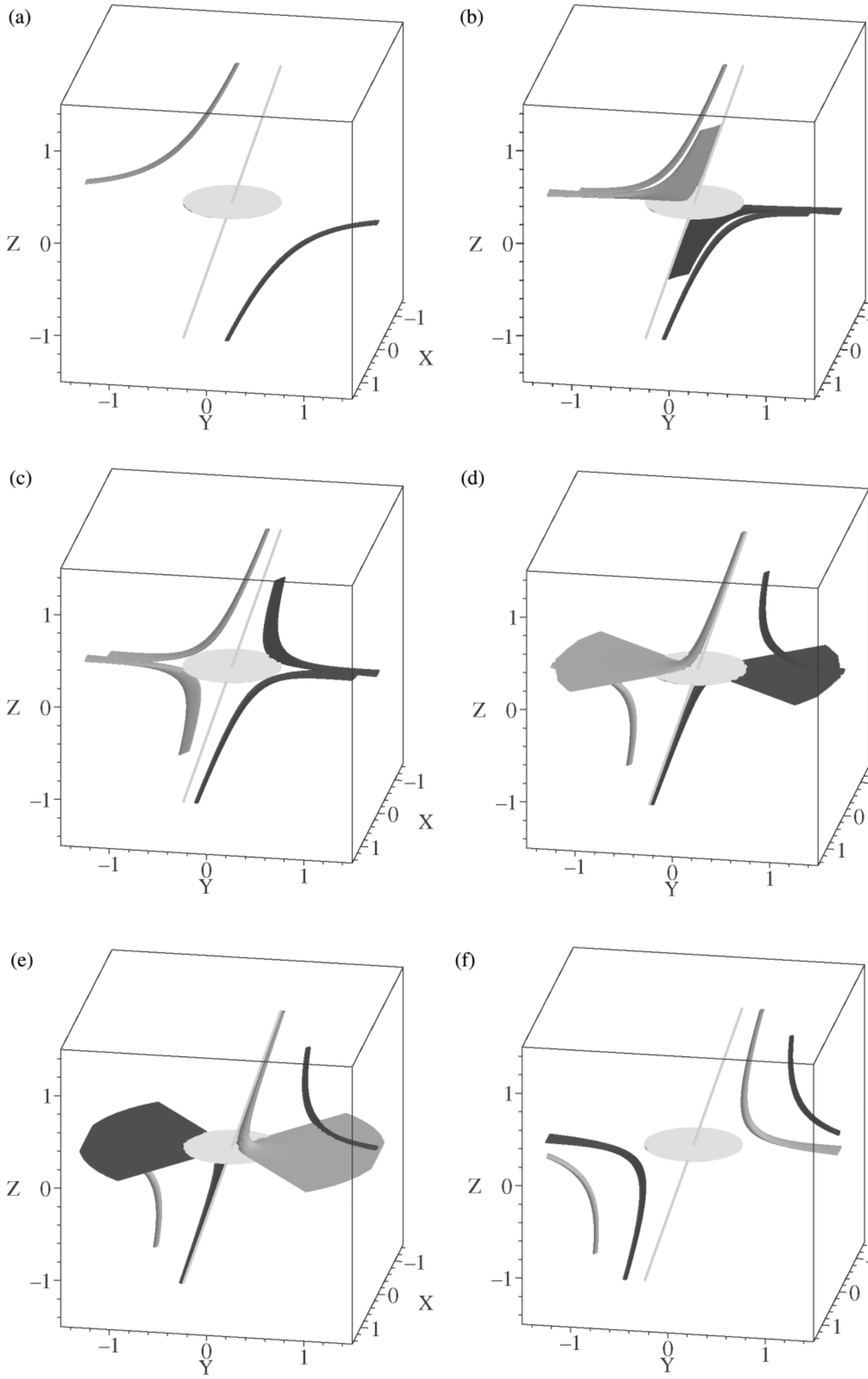

Figure 5. Sequence of snapshots showing the reconnection of two thin flux tubes, initially chosen to be symmetric about the null. Field lines traced from footpoints anchored in the fan-crossing flow flip up the spine [(b)–(c)]. Field lines traced from the top/bottom of the domain (anchored in the spine-crossing flow) flip around the spine in the fan plane [(d)–(f)]. The grey line is the spine and the grey disc shows the location of the fan plane.

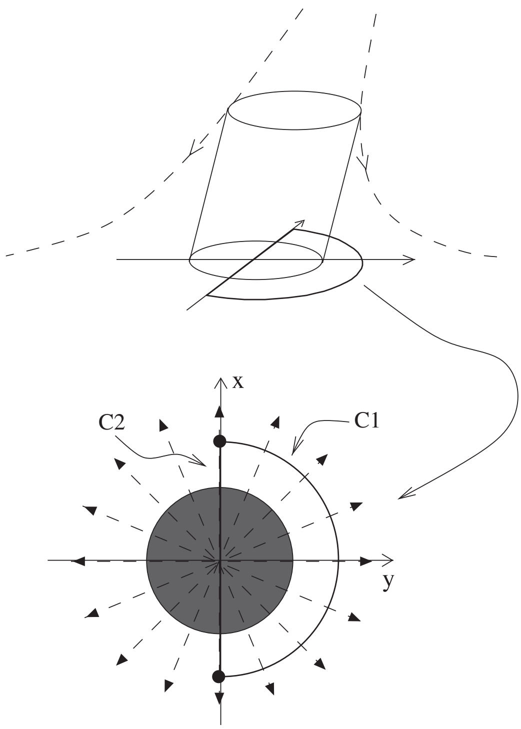

Figure 6. The curves $C1$ and $C2$ joining two points on the x-axis, on opposite sides of the diffusion region, in the fan plane. The curve $C1$ is semi-circular and therefore perpendicular to **B**.

It is possible to calculate the amount of flux transported across the fan surface by evaluating

$$F = \int_{C1} \mathbf{v} \times \mathbf{B} \cdot \mathrm{d}\mathbf{l}, \tag{18}$$

where $C1$ is the curve shown in figure 6, which lies in the fan plane. Note that, since the flux is transported across the fan in opposite directions for positive and negative y, the total amount transported across the fan plane is double this amount, with equal amounts transported in either direction.

Now, since the curve $C1$ lies outside D, and therefore along it $\mathbf{v} \times \mathbf{B} = -\mathbf{E}$, we can write $F = -\int_{C1} \mathbf{E} \cdot \mathrm{d}\mathbf{l}$, and since $\mathbf{E}$ is conservative this integral is independent of the path, so we may write

$$F = -\int_{C2} E_{\parallel} \mathrm{dl}, \tag{19}$$

where the curve $C2$, as shown in figure 6, runs along a pair of field lines coincident with the x-axis. Due to the form of equation (19), and the fact that $C2$ passes through the null, we define F as the reconnection rate. From equation (19), we have

$$F = -\int_{-a}^{a} E_x \, \mathrm{d}x = \frac{16}{15} B_0 \, j \eta_0 \, a. \tag{20}$$

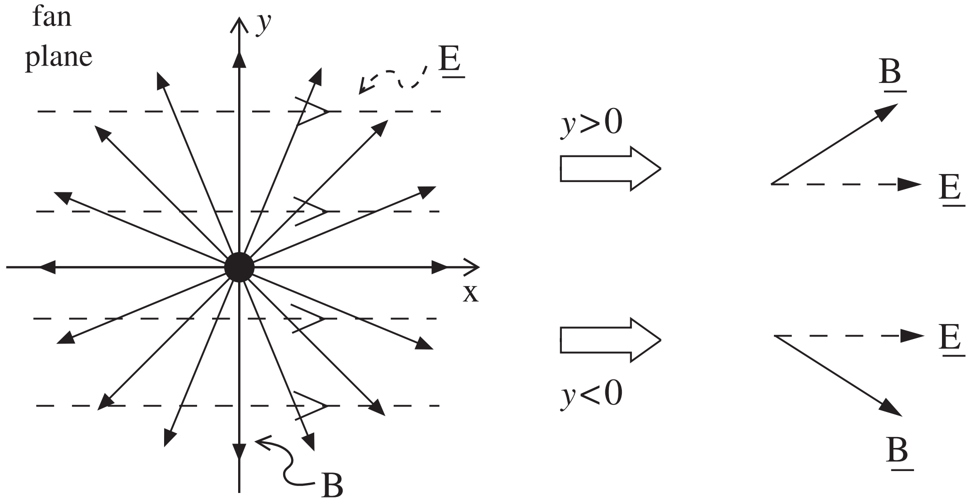

Figure 7. The structure of the magnetic field, **B**, and electric field, **E**, in the fan plane. The non-zero vector product between the two implies a flow across the fan, in opposite directions for $y > 0$ and $y < 0$.

The physical interpretation of this reconnection rate is as follows. Firstly, it is similar to the reconnection rate of Paper I, in that it is calculated by integrating the parallel electric field along the direction parallel to **J**. Moreover, there are also similarities to the case of 2D reconnection with an invariant third direction, since the reconnection rate gives a measure of the amount of flux transported across the separatrix surface(s) of the null.

The plasma flow across the fan plane is very important for topological considerations, and is a major difference between this solution and the one described in Paper I. The cause of this fan-crossing flow can be understood as follows. Consider simply the structure of **B** and **J**, in the vicinity of the x-axis, in the fan plane. $\mathbf{J}\cdot\mathbf{B}$ has opposite sign for $x > 0$ and $x < 0$, since **J** and **B** are parallel on the x-axis for $x > 0$ and anti-parallel for $x < 0$. So $\Phi = -\int \eta \mathbf{J}\cdot\mathbf{B}\,\mathrm{d}s$ has opposite sign for $x > 0$ and $x < 0$. Hence, in the fan plane close to the x-axis, $\mathbf{E} = (E_x, 0, 0)$ is unidirectional across the spine, as shown in figure 7. Since $\mathbf{E}\times\mathbf{B}$ is non-zero in the fan plane, we get a plasma flow in the z-direction, across the fan. Note that v_z must have different signs for $y > 0$ and $y < 0$ due to the different handedness of the vector products in these two regions (see figure 7). Note that this argument is completely independent of the profile of η, and relies only upon the structure of **B** (and thus **J**) and the fact that η is localised.

5. Non-existence of perfectly reconnecting field lines

A major result of Paper I and also of Horing and Priest (2003) was that in three dimensions there is no simple one-to-one correspondence of reconnecting field lines. That is, for any field line which undergoes reconnection, there is in general no corresponding field line which upon reconnection will have both its footpoints mapping to the footpoints of a another field line. Consequently, there is no one-to-one correspondence, or perfect matching, of reconnecting flux tubes in general.

It turns out that, by tracking the change of connections of field lines for our solution, it can be shown that once again no one-to-one field line reconnection occurs. This can be shown explicitly by following the procedure shown in figure 8. To find reconnecting field lines we consider starting from an initial point (or plasma element) outside the diffusion region on a given field line. Start from a point close to the spine axis, in the 'first quadrant', as shown in figure 8. We allow this point to evolve in the ideal flow for a time $t = t'$, where t' is large enough that the plasma element crosses the spine. On the new field line (in quadrant 2) on which our plasma element now lies, choose a point (outside D) in the fan-crossing region of the flow and follow this point backwards in $\mathbf{v}$ from $t = t'$ to $t = 0$ to find the reconnecting field line in quadrant 3. Now repeat these two steps. Note that performing this procedure is analogous to performing a step of the iteration procedure described by Horing and Priest (2003). In this case, the procedure is performed in the constant-x plane (shown in figure 8) which is the analogue of the constant-z plane, also perpendicular to $\mathbf{J}$, in Horing and Priest (2003).

If field lines reconnected in a one-to-one fashion, then after completing the circuit and returning to quadrant 1, we would return to a point lying on the initial field line. However, performing this routine for our solution we find that we never end up back on the same field line, and hence there is no field line with which our initial magnetic field line will perfectly reconnect, such that their footpoints become pair-wise oppositely connected after the reconnection process.

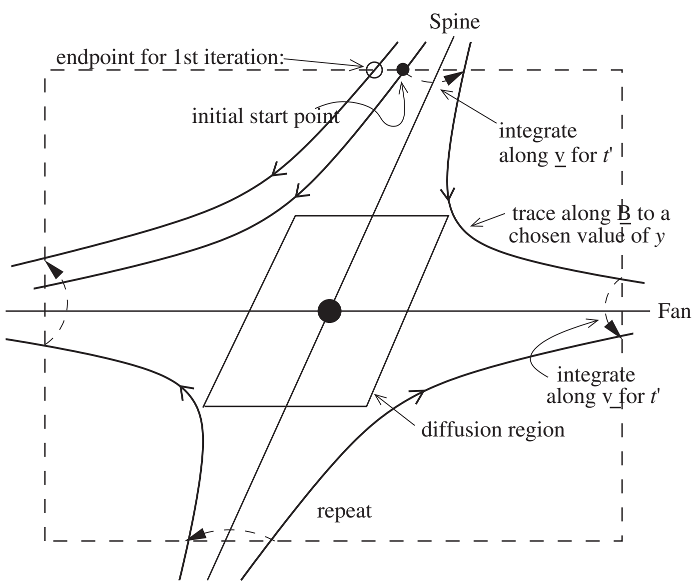

Figure 8. Circuit of magnetic field lines (solid black) and plasma flow lines followed by the corresponding field line footpoints (dashed black). In general the start and end points of the circuit lies on different field lines, implying no one-to-one correspondence of reconnecting field lines. We define quadrant 1 as the top left area bounded by the fan and spine, and number the other quadrants clockwise.

6. Non-constant current

It is possible to perform the same analytical analysis as described in section 3 for certain slightly more complex magnetic fields, whose associated currents, **J**, are not constant. It is instructive, in particular, to consider situations in which the current vanishes at the null point itself. We consider first the magnetic field

$$\mathbf{B} = B_0\big(x, y - jz^3, \; -2z\big), \tag{21}$$

so that the current is now $\mathbf{J} = (B_0/\mu_0)(3jz^2, 0, 0)$. The fan of the null point again lies in the $z = 0$ plane, although now the spine curves away from the z-axis. We note that the current is zero at $z = 0$, in the fan.

Repeating the analysis described in section 3, with the profile of η again desribed by equation (16), where this time $R_1^2 = x^2 + (y - jz^3/7)^2$, we obtain the plasma flow shown in figure 9. The flow once again crosses the spine of the null point, but this time there is no flow across the fan, $v_z(z = 0) = 0$. The effect on the reconnection of magnetic field lines is that we have a slippage-type behaviour of field lines which pass through the diffusion region. Both ends of a given field line, passing through D, move, outside D, at the ideal plasma velocity. This plasma velocity is, as shown in figure 9, largely in the y-direction, and thus the field line is moved through D in this direction, with each 'half' of the field line slipping relative to the other during the time they are in D. The result is that, after leaving D, the two footpoints are differently connected, but continue to move in the same direction. Note also that, by analogy with our previous argument (considering the structure of **B** and **J** in the fan), it is no surprise that we now have no flow across the fan, since **J** is now zero in the fan plane.

The motion of the magnetic field lines under the process described above is very similar to the field line behaviour associated with the fan reconnection of Priest and

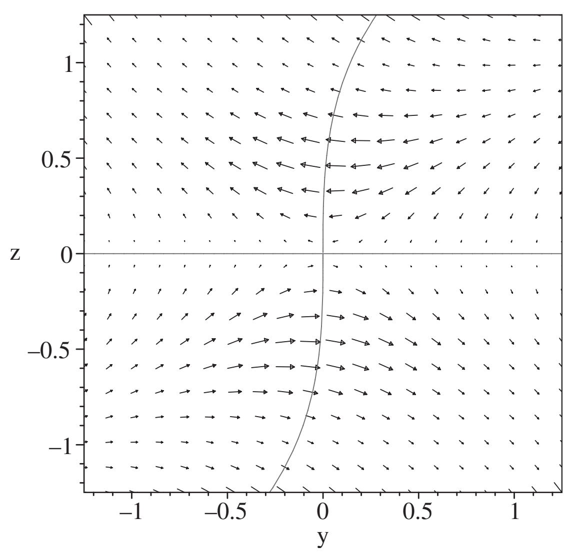

Figure 9. The plasma flow and magnetic skeleton in a plane of constant x when $\mathbf{J} \propto (z^2, 0, 0)$, for the same choice of parameters as figure 4.

Titov (1996). This behaviour is obtained by considering a situation in which the electric current is zero in the fan plane of the null point. Is it possible, then, to find a configuration in which the field line behaviour is analogous to *spine reconnection*, such that there is a plasma flow across the fan of the null point but not the spine? In order to answer this question, we consider a null point where the current is zero on the spine.

Consider the magnetic field

$$\mathbf{B} = B_0\left(x, y, -2z + \tfrac{1}{3}jy^3\right), \tag{22}$$

whose spine lies along the z-axis, and whose fan is the surface $z = jy^3/15$. The resultant current is given by $\mathbf{J} = B_0/\mu_0(jy^2, 0, 0)$ which again lies parallel to the fan plane. We repeat once more the analysis previously described, where this time η is prescribed, for analytical simplicity, as

$$\eta = \eta_0 \begin{cases} \left[\left(\dfrac{R}{a}\right)^2 - 1\right]^2 \left[\left(\dfrac{z_1}{b}\right)^2 - 1\right]^2 & R < a, \quad {z_1}^2 < b^2 \\ 0 & \text{otherwise,} \end{cases}, \tag{23}$$

where $R = \sqrt{x^2 + y^2}$ and $z_1 = z - jy^3/15$. The resulting plasma flow in this case is shown for a typical plane of constant x in figure 10a. As can be seen, there is a plasma flow in this case across the fan, but not across the spine, and so we have a field line behaviour similar to the spine reconnection of Priest and Titov (1996), where flux is continually transported across the fan of the null point. However, the nature of the flow across the fan is not as simple in this case as in the constant-current solution. In the constant-current solution the plasma velocity is qualitatively the same for all planes of constant x, and so there is little variation in the nature of the reconnection process. Flux is simply transported across the fan in one direction for $y > 0$ and the other direction for $y < 0$. However, when we move to this situation where $\mathbf{J} \propto (y^2, 0, 0)$, the structure is not quite so simple. The difference can be clearly seen by considering surfaces of Φ in the fan plane for each solution (see figure 10). In the constant current solution, Φ decreases monotonically along, as well as away from, the x-axis (see figure 10b), resulting in the simple flux transport described. However, when $\mathbf{J} \propto (y^2, 0, 0)$, the potential, Φ, is zero along the x-axis, and there are two positive peaks and two negative (see figure 10c). (The disparity between the peak Φ values for the two solutions is due to the extra term required in the constant-current solution to ensure smooth variation of physical quantities across the fan plane.) As a result, there are different regions where flux is transported back and forth across the fan in opposite directions.

This reconnection process has two different possible interpretations. Consider performing the same analysis as performed in section 4 in order to find the reconnection rate. Performing the integration $F = -\int_{-a}^{a} E_x \mathrm{d}x$, should again provide us with a reconnection rate, with physical interpretation being the rate of flux transport across the fan. However, in this case, evaluating this integral we obtain the answer zero. Alternatively, the integral (18) can instead be evaluated around only a quarter circle, to give $4\sqrt{2}\eta_0 B_0 j a^3$. The reconnecion rate can thus be thought of as zero, due to two reconnection processes cancelling each other, or as $16\sqrt{2}\eta_0 B_0 j a^3$, if the two processes are envisaged to complement each other.

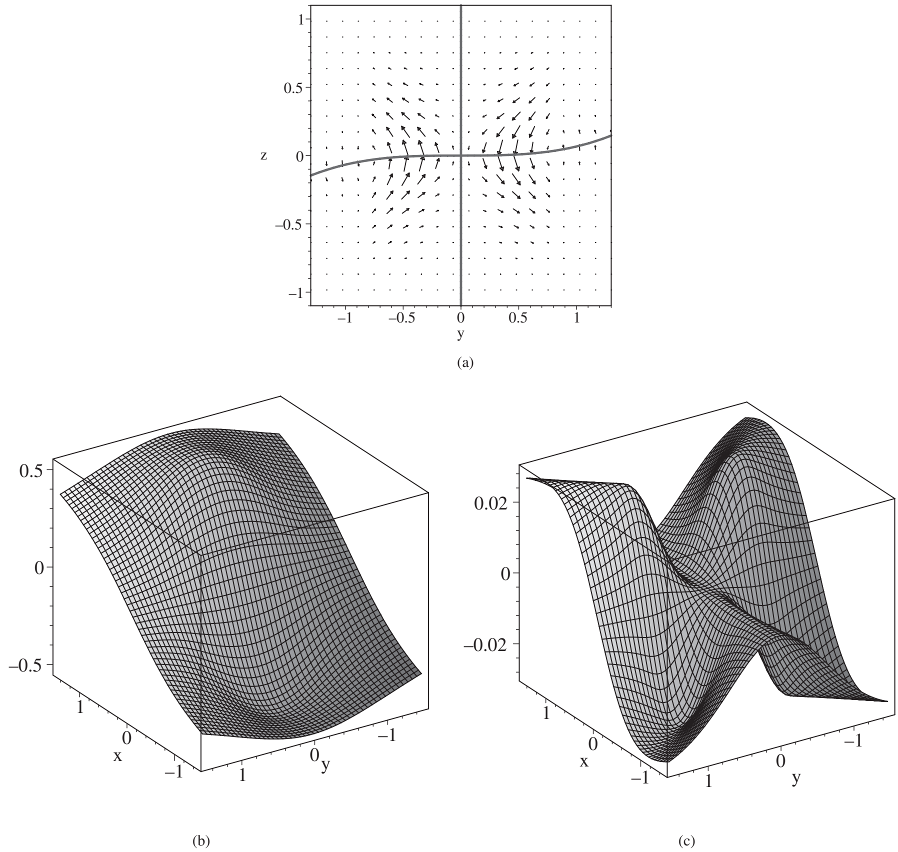

Figure 10. The plasma flow and magnetic skeleton in a plane of constant $x \neq 0$ when $\mathbf{J} \propto (y^2, 0, 0)$, for the same choice of parameters as figure 4. (b) Surface of the electric potential, Φ, in the fan plane for the constant current solution and (c) for the solution with $\mathbf{J} \propto (y^2, 0, 0)$.

7. Conclusions

We have described here steady solutions of the kinematic resistive MHD equations, which exist in the vicinity of a magnetic null point with fan-aligned current, with a diffusion term localised around the null. In contrast to what was found in Paper I, the result of the fan-aligned current is that flows are present across both the spine line and the fan plane of the null. The flow across the fan plane is of particular interest, since in a global field, for example in the solar corona, the fan surface is a separatrix of the field, and thus a transfer of flux across it constitutes a change in the magnetic topology. A further solution is found in which the current drops to zero at the null point itself, resulting in flow across the fan only, as well as a solution with a solely spine-crossing flow. In general, one would expect to have more general types of currents and therefore plasma flows, and so a general situation would exhibit facets of both the behaviour described here, and that described in Paper I. The situations discussed in the two papers, where the current is exactly aligned with either the spine or fan of the null, are of course special cases.

Although the nature of the reconnection discovered here is very different from the case where the current lies parallel with the spine (see Paper I), a number of key properties are still present. In particular, no unique field line velocity exists, and so the process must be descibed by two separate flux velocities, $\mathbf{w}_{in}$ and $\mathbf{w}_{out}$, which do not coincide within the non-ideal region. Furthermore, it is found that there is no simple one-to-one correspondence of reconnecting field lines after reconnection has occurred. The result is that the re-ordering of flux by the reconnection process is much more complicated than at a familiar two-dimensional null point.

In the future, we plan to set up a numerical experiment for the full equations, including the equation of motion, and incorporating the invaluable insights that we have gained from this paper. In particular, we shall calculate the flux motions $\mathbf{w}_{in}$ and $\mathbf{w}_{out}$ and will investigate whether the different types of kinematic reconnection that we have discovered here and in previous papers are realisable in the dynamic regime.

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